

Finite BRST-antiBRST Transformations in Generalized Hamiltonian Formalism

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October 16, 2014

Abstract

We introduce the notion of finite BRST-antiBRST transformations for constrained dynamical systems in the generalized Hamiltonian formalism, both global and field-dependent, with a doublet λ_a , $a = 1, 2$, of anticommuting Grassmann parameters and find explicit Jacobians corresponding to these changes of variables in the path integral. It turns out that the finite transformations are quadratic in their parameters. Exactly as in the case of finite field-dependent BRST-antiBRST transformations for the Yang–Mills vacuum functional in the Lagrangian formalism examined in our previous paper [arXiv:1405.0790[hep-th]], special field-dependent BRST-antiBRST transformations with functionally-dependent parameters $\lambda_a = \int dt (s_a \Lambda)$, generated by a finite even-valued function $\Lambda(t)$ and by the anticommuting generators s_a of BRST-antiBRST transformations, amount to a precise change of the gauge-fixing function for arbitrary constrained dynamical systems. This proves the independence of the vacuum functional under such transformations. We derive a new form of the Ward identities, depending on the parameters λ_a , and study the problem of gauge-dependence. We present the form of transformation parameters which generates a change of the gauge in the Hamiltonian path integral, evaluate it explicitly for connecting two arbitrary R_ξ -like gauges in the Yang–Mills theory and establish, after integration over momenta, a coincidence with the Lagrangian path integral [arXiv:1405.0790[hep-th]], which justifies the unitarity of the S -matrix in the Lagrangian approach.

Keywords: constrained dynamical systems, BRST-antiBRST generalized Hamiltonian quantization, field-dependent BRST-antiBRST transformations, Yang–Mills theory

1 Introduction

It is well known that modern quantization methods for gauge theories in the Lagrangian and Hamiltonian formulations [1, 2, 3, 4] are based mainly on the principles of BRST symmetry [5, 6, 7] and BRST-antiBRST symmetry [8, 9, 10], which are characterized by the presence of one Grassmann-odd parameter μ and two Grassmann-odd parameters $(\mu, \bar{\mu})$, respectively. The parameters of the $\text{Sp}(2)$ -covariant generalized Hamiltonian [11, 12] and Lagrangian [14, 15] quantization schemes (see also [13, 16]) form an $\text{Sp}(2)$ -doublet: $(\mu, \bar{\mu}) \equiv (\mu_1, \mu_2) = \mu_a$. These parameters were initially considered as infinitesimal odd-valued objects and may be regarded as constants and as field-dependent functionals,

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used, respectively, to obtain the Ward identities and to establish the gauge-independence of the corresponding vacuum functional in the path integral approach.

In our recent work [17], we have suggested an extension of BRST-antiBRST transformations in Lagrangian formalism to finite (both global and field-dependent) parameters in Yang–Mills and general gauge theories, which in the latter case has been recently developed in [18, 19]. The idea of “finiteness” is also based on the inclusion into BRST-antiBRST transformations of a new term, being quadratic in the transformation parameters λ_a . This makes it possible to realize the complete BRST-antiBRST invariance of the integrand in the vacuum functional. The functionally-dependent parameters $\lambda_a = s_a \Lambda$, induced by a Grassmann-even functional Λ , provide an explicit correspondence (due to a so-called compensation equation for the corresponding Jacobian) between the choices of Λ connecting the partition function of a theory in a certain gauge (determined by a gauge Bosonic functional F_0) with the theory in a different gauge, given by another gauge Boson F . This becomes a key instrument to determine, in a BRST-antiBRST approach, the Gribov horizon functional [20] – given by the Landau gauge in the Gribov–Zwanziger theory [21] – by using any other gauge, including the R_ξ -gauges, which eliminate residual gauge invariance in the deep IR region. Notice that the finite BRST-antiBRST transformations are, in fact, constructed from infinitesimal gauge transformations (instead of finite gauge group transformations) of classical variables in the case of finite values of gauge parameters. Therefore, finite BRST-antiBRST transformations developed within perturbative theory may be used to consistently¹ determine the Gribov horizon functional in any differential gauge (due to Singer’s result [22]), starting from the horizon functional in a fixed gauge, which, in turn, should be obtained non-perturbatively from finite gauge group transformations.

For the sake of completeness, let us remind that finite field-dependent BRST transformations were introduced [23] in the Yang–Mills theory (with the quantum action constructed by the Faddeev–Popov rules [24]), on the basis of a functional equation for the parameter used to provide the path integral with a change of variables that would allow one to relate the quantum action in a certain gauge with the quantum action in a different gauge. This equation and a similar equation [25] for the finite parameter of a field-dependent BRST transformation in generalized Hamiltonian formalism were solved in a series of particular cases for parameters; however, a general solution was not presented.

The recent studies [26, 27] have proposed the idea of finite BRST–BFV transformations [26] in the generalized Hamiltonian formalism [7, 28, 29], as well as finite BRST [27] and BRST–BV [30] transformations, using different path integral representations in the Batalin–Vilkovisky (BV) formalism [31]. It has been shown that, in order to relate partition functions given by different gauges, it is sufficient to solve a compensation equation for the corresponding finite field-dependent parameter, first suggested in [32] for Yang–Mills theories in the Faddeev–Popov procedure [24]. This problem was raised in [33] to explore the issue of gauge-independence in gauge theories with so-called soft breaking of BRST symmetry, which is related to a consistent construction of the Gribov horizon functional [21] by using different gauges [34, 35].

Thus, the problem of setting up a construction of finite BRST-antiBRST transformations for arbitrary dynamical systems with first-class constraints and investigating its properties in generalized Hamiltonian formalism is open even in Yang–Mills theories. This problem is related to establishing a correspondence of the quantum action in the BRST-antiBRST generalized Hamiltonian quantization [11, 12] – where gauge is introduced by a Bosonic gauge-fixing function of phase-space variables Φ – with the quantum action of the same theory in a different gauge $\Phi + \Delta\Phi$ for a finite value of $\Delta\Phi$, by using a change of variables in the path integral.

Based on these reasons, we intend to address the following issues in the case of dynamical systems with first-class constraints in the generalized Hamiltonian formalism:

1. introduction of *finite BRST-antiBRST transformations*, being polynomial in powers of a constant $\text{Sp}(2)$ -doublet of Grassmann-odd parameters λ_a and leaving the integrand in the Hamiltonian path integral for vanishing

¹Namely, in a way that preserves the gauge-independence of the physical S -matrix.

external sources invariant to all orders in λ_a ;

2. definition of *finite field-dependent BRST-antiBRST transformations* as polynomials in the $\text{Sp}(2)$ -doublet of Grassmann-odd functionals $\lambda_a(\Gamma)$ depending on the entire set of symplectic coordinates of the total phase space; calculation of the Jacobian related to this change of variables by using a special class of transformations with s_a -potential parameters $\lambda_a(\Gamma) = \int dt s_a \Lambda(\Gamma(t))$, for a Grassmann-even function $\Lambda(\Gamma(t))$ and Grassmann-odd generators s_a of BRST-antiBRST transformations;
3. construction of a solution to the compensation equation for an unknown function Λ generating the $\text{Sp}(2)$ -doublet λ_a to establish a relation of the Hamiltonian action $S_{H,\Phi}$ in a certain gauge determined by a gauge Boson Φ with the Hamiltonian action $S_{H,\Phi+\Delta\Phi}$ in a different gauge $\Phi + \Delta\Phi$;
4. explicit construction of the parameters λ_a of finite field-dependent BRST-antiBRST transformations generating a change of the gauge in the Hamiltonian path integral within a class of linear R_ξ -like gauges in the Hamiltonian formalism, which are realized in terms of Bosonic gauge functions $\Phi_{(\xi)}$, with $\xi = 0, 1$ corresponding to the Landau and Feynman (covariant) gauges, respectively.

The work is organized as follows. In Section 2, we remind the general setup of the BRST-antiBRST generalized Hamiltonian quantization of dynamical systems with first-class constraints and list its basics ingredients. In Section 3, we introduce the notion of finite BRST-antiBRST transformations with constant and field-dependent parameters in generalized Hamiltonian formalism. We obtain explicit Jacobians corresponding to these changes of variables and show that, exactly as in the case of field-dependent BRST-antiBRST transformations for the Yang–Mills vacuum functional [17] in Lagrangian formalism, the corresponding field-dependent transformations amount to a precise change of the gauge-fixing functional. Here, we also study the group properties of finite field-dependent BRST-antiBRST transformations. In Section 4, we derive the Ward identities with the help of field-dependent BRST-antiBRST transformations and study the gauge dependence of the generating functionals of Green’s functions. In Section 5, we present the form of transformation parameters that generates a change of the gauge and evaluate it for connecting two arbitrary R_ξ -like gauges in Yang–Mills theories. In Conclusion, we discuss the results and outline some open problems. In Appendix A, we present a detailed calculation of the Jacobians corresponding to the finite BRST-antiBRST Hamiltonian transformations with constant and field-dependent parameters.

We use condensed notations similar to [36], namely, the spatial coordinates of canonical field variables $\Gamma^p = (P_A, Q^A)$ are absorbed into the indices p, A , whereas integration over the spatial coordinates is included into summation over repeated indices. The partial $\partial/\partial\Gamma^p$ and variational $\delta/\delta\Gamma^p$ derivatives over Γ^p are understood as acting from the right. The variational derivative $\delta/\delta\Gamma^p(t)$ is taken along a phase-space trajectory $\Gamma^p(t)$, whereas the partial derivative $\partial/\partial\Gamma^p$ of a field variable Γ^p is understood as the variational derivative with fixed time, $\delta_t/\delta\Gamma^p$, as in [3], applied to a functional $\mathcal{F}(\Gamma(t))$ local in time, $\delta\mathcal{F} = (\delta_t\mathcal{F}/\delta\Gamma^p)\delta\Gamma^p$, $\delta_t/\delta\Gamma^p \equiv \partial/\partial\Gamma^p$. We refer to t -local functionals $\mathcal{F}(\Gamma)$ as *functions*, whereas the corresponding $F(\Gamma) = \int dt \mathcal{F}(\Gamma(t))$ are called *functionals*. The raising and lowering of $\text{Sp}(2)$ indices, $s^a = \varepsilon^{ab}s_b$, $s_a = \varepsilon_{ab}s^b$, is carried out with the help of a constant antisymmetric second-rank tensor ε^{ab} , $\varepsilon^{ac}\varepsilon_{cb} = \delta_b^a$, subject to the normalization condition $\varepsilon^{12} = 1$. The Grassmann parity and ghost number of a quantity A , assumed to be homogeneous with respect to these characteristics, are denoted by $\varepsilon(A)$, $\text{gh}(A)$, respectively. By default, we understand BRST-antiBRST transformations in generalized Hamiltonian formalism as *infinitesimal* invariance transformations with a doublet λ_a of anticommuting parameters, whereas *finite BRST-antiBRST transformations* are understood as transformations of invariance to all powers of the transformation parameters λ_a .

2 Basics of BRST-antiBRST Generalized Hamiltonian Quantization

We recall that the total phase space underlying the BRST-antiBRST generalized Hamiltonian quantization is parameterized by the canonical phase-space variables, Γ^p , $\varepsilon(\Gamma^p) = \varepsilon_p$,

$$\Gamma^p = (P_A, Q^A) = (\eta, \Gamma_{\text{gh}}) , \quad (2.1)$$

where $\eta = (p_i, q^i)$ are the classical momenta and coordinates of a given dynamical system, described by a Hamiltonian $H_0 = H_0(\eta)$ and by a set of (generally, linearly dependent) first-class constraints $T_{\alpha_0} = T_{\alpha_0}(\eta)$, $\varepsilon(T_{\alpha_0}) = \varepsilon_{\alpha_0}$, subject to involution relations in terms of the Poisson superbracket at a fixed time instant t , $\{\Gamma^p, \Gamma^q\} = \omega^{pq} = \text{const}$, with ω^{pq} being an even supermatrix, $\omega^{pq} = -(-1)^{\varepsilon_p \varepsilon_q} \omega^{qp}$,

$$\{H_0, T_{\alpha_0}\} = T_{\gamma_0} V_{\alpha_0}^{\gamma_0}, \quad \{T_{\alpha_0}, T_{\beta_0}\} = T_{\gamma_0} U_{\alpha_0 \beta_0}^{\gamma_0}, \quad \text{for} \quad U_{\alpha_0 \beta_0}^{\gamma_0} = -(-1)^{\varepsilon_{\alpha_0} \varepsilon_{\beta_0}} U_{\beta_0 \alpha_0}^{\gamma_0} . \quad (2.2)$$

The variables Γ_{gh} in (2.1) contain the entire set of auxiliary variables that correspond to the towers [28] of ghost-antighost coordinates C and Lagrangian multipliers π , as well as their respective conjugate momenta \mathcal{P} and λ , arranged within the BRST-antiBRST generalized Hamiltonian quantization [11, 12] into $\text{Sp}(2)$ -symmetric tensors for an L -th stage of reducibility ($L = 0$ corresponding to an irreducible theory),

$$\Gamma_{\text{gh}} = \left(\mathcal{P}_{\alpha_s | a_0 \dots a_s}, \quad C^{\alpha_s | a_0 \dots a_s}, \quad \lambda_{\alpha_s | a_1 \dots a_s}, \quad \pi^{\alpha_s | a_1 \dots a_s}, \quad s = 0, 1, \dots, L \right) ,$$

with the corresponding distribution [12] of the Grassmann parity and ghost number.

The generating functional of Green's functions for a dynamical system in question has the form

$$Z_{\Phi}(I) = \int d\Gamma \exp \left\{ \frac{i}{\hbar} \int dt \left[\frac{1}{2} \Gamma^p(t) \omega_{pq} \dot{\Gamma}^q(t) - H_{\Phi}(t) + I(t) \Gamma(t) \right] \right\} \quad (2.3)$$

and determines the partition function $Z_{\Phi} = Z_{\Phi}(0)$ at the vanishing external sources $I_p(t)$ to Γ^p . In (2.3), integration over time is taken over the range $t_{\text{in}} \leq t \leq t_{\text{out}}$; the functions of time $\Gamma^p(t) \equiv \Gamma_t^p$ for $t_{\text{in}} \leq t \leq t_{\text{out}}$ are trajectories, $\dot{\Gamma}^p(t) \equiv d\Gamma^p(t)/dt$; the quantities $\omega_{pq} = (-1)^{(\varepsilon_p+1)(\varepsilon_q+1)} \omega_{qp}$ compose an even supermatrix inverse to that with the elements ω^{pq} ; the unitarizing Hamiltonian $H_{\Phi}(t) = H_{\Phi}(\Gamma(t))$ is determined by four t -local functions: $\mathcal{H}(t)$, an $\text{Sp}(2)$ -doublet of odd-valued functions $\Omega^a(t)$, with $\text{gh}(\Omega^a) = -(-1)^a$, and an even-valued function $\Phi(t)$, with $\text{gh}(\Phi) = 0$, known as the gauge-fixing Boson, which are given by the equations

$$H_{\Phi}(t) = \mathcal{H}(t) + \frac{1}{2} \varepsilon_{ab} \{ \{ \Phi(t), \Omega^a(t) \}_t, \Omega^b(t) \}_t, \quad \text{with} \quad \{ A(t), B(t) \}_t = \{ A(\Gamma), B(\Gamma) \}|_{\Gamma=\Gamma(t)}, \quad \text{for any } A, B, \quad (2.4)$$

$$\{ \Omega^a, \Omega^b \} = 0, \quad \{ \mathcal{H}, \Omega^b \} = 0, \quad (2.5)$$

with the boundary conditions

$$\mathcal{H}|_{\Gamma_{\text{gh}}=0} = H_0(\eta), \quad \left. \frac{\delta \Omega^a}{\delta C^{\alpha_0 b}} \right|_{\Gamma_{\text{gh}}=0} = \delta_b^a T_{\alpha_0}(\eta). \quad (2.6)$$

From equations (2.5) and the Jacobi identities for the Poisson superbracket, it follows that

$$\{ H_{\Phi}, \Omega^a \} = 0. \quad (2.7)$$

The integrand in (2.3) is invariant with respect to the infinitesimal BRST-antiBRST transformations [11]

$$\Gamma^p \rightarrow \check{\Gamma}^p = \Gamma^p + (s^a \Gamma^p) \mu_a, \quad \text{with} \quad s^a = \{ \bullet, \Omega^a \}, \quad (2.8)$$

realized on phase-space trajectories $\Gamma^p(t)$ as

$$\Gamma^p(t) \rightarrow \check{\Gamma}^p(t) = \Gamma^p(t) + \{ \Gamma^p(t), \Omega^a(t) \}_t \mu_a = \Gamma^p(t) + (s^a \Gamma^p)(t) \mu_a, \quad (2.9)$$

with an $\text{Sp}(2)$ -doublet μ_a of anticommuting constant infinitesimal parameters, $\mu_a \mu_b + \mu_b \mu_a \equiv 0$, for any $a, b = 1, 2$. The generators s^a of BRST-antiBRST transformations are anticommuting, nilpotent and obey the Leibnitz rule when acting on the product and the Poisson superbracket:

$$s^a s^b + s^b s^a = 0, \quad s^a s^b s^c = 0, \quad s^a (AB) = (s^a A) B (-1)^{\varepsilon_B} + A (s^a B), \quad s^a \{A, B\} = \{s^a A, B\} (-1)^{\varepsilon_B} + \{A, s^a B\}. \quad (2.10)$$

The BRST-antiBRST invariance of the integrand in (2.3) with $I_p(t) = 0$ under the transformations (2.9) allows one to obtain the Ward identities for $Z_\Phi(I)$, namely,

$$\langle \int dt I_p(t) s^a \Gamma^p(t) \rangle_{\Phi, I} = 0, \quad (2.11)$$

$$\text{for } \langle \mathcal{O} \rangle_{\Phi, I} = Z_\Phi^{-1}(I) \int d\Gamma \mathcal{O} \exp \left\{ \frac{i}{\hbar} \left[S_{H, \Phi}(\Gamma) + \int dt I_p(t) \Gamma^p(t) \right] \right\},$$

$$\text{with } S_{H, \Phi}(\Gamma) = \int dt \left[\frac{1}{2} \Gamma^p(t) \omega_{pq} \dot{\Gamma}^q(t) - H_\Phi(t) \right], \quad (2.12)$$

where the expectation value of a functional $\mathcal{O}(\Gamma)$ is calculated with respect to a certain gauge $\Phi(\Gamma)$ in the presence of external sources I_p . To obtain (2.11), we subject (2.3) to a change of variables $\Gamma \rightarrow \Gamma + \delta\Gamma$ with $\delta\Gamma$ given by (2.9) and use the equations (2.7) for $H(t)$. At the same time, with allowance for the equivalence theorem [37], the transformations (2.9) allow one to establish the independence of the S -matrix from the choice of a gauge. Indeed, if we change the gauge, $\Phi \rightarrow \Phi + \Delta\Phi$, by an infinitesimal value $\Delta\Phi$ in Z_Φ and make the change of variables (2.9), choosing the parameters μ_a as functionals of Γ^p (i.e., not as functions of time t or of the variables Γ^p), namely,

$$\mu_a = \frac{i}{2\hbar} \varepsilon_{ab} \int dt \{ \Delta\Phi, \Omega^b \}_t = \frac{i}{2\hbar} \int dt (s_a \Delta\Phi)(t), \quad (2.13)$$

we arrive at $Z_{\Phi+\Delta\Phi} = Z_\Phi$, and therefore the S -matrix is gauge-independent.

3 Finite BRST-antiBRST Transformations

In this section, we introduce (Subsection 3.1) the notion of finite BRST-antiBRST transformations and examine two classes of such transformation, namely, those with constant and field-dependent parameters, each class being realized in a t -local form and in a functional form. We calculate (Subsection 3.2) the corresponding Jacobians, derive (Subsection 3.3) the compensation equation and present its solution. Finally, we study (Subsection 3.4) some group properties of field-dependent BRST-antiBRST transformations.

3.1 Definitions

Let us introduce finite transformations of the canonical variables Γ^p with a doublet λ_a of anticommuting Grassmann parameters, $\lambda_a \lambda_b + \lambda_b \lambda_a = 0$,

$$\Gamma^p \rightarrow \check{\Gamma}^p = \Gamma^p + \Delta\Gamma^p = \check{\Gamma}^p(\Gamma|\lambda), \quad \text{so that } \check{\Gamma}^p(|0) = \Gamma^p. \quad (3.1)$$

In general, such transformations are quadratic in their parameters, due to $\lambda_a \lambda_b \lambda_c \equiv 0$,

$$\check{\Gamma}^p(\Gamma|\lambda) = \check{\Gamma}^p(\Gamma|0) + \left[\check{\Gamma}^p(\Gamma|\lambda) \frac{\overleftarrow{\partial}}{\partial \lambda_a} \right]_{\lambda=0} \lambda_a + \frac{1}{2} \left[\check{\Gamma}^p(\Gamma|\lambda) \frac{\overleftarrow{\partial}}{\partial \lambda_a} \frac{\overleftarrow{\partial}}{\partial \lambda_b} \right] \lambda_b \lambda_a, \quad (3.2)$$

which implies

$$\Delta\check{\Gamma}^p = Z^{pa} \lambda_a + (1/2) Z^p \lambda^2, \quad \text{where } \lambda^2 \equiv \lambda_a \lambda^a, \quad (3.3)$$

for certain functions $Z^{pa} = Z^{pa}(\Gamma)$, $Z^p = Z^p(\Gamma)$, corresponding to the first- and second-order derivatives of $\check{\Gamma}^p(\Gamma|\lambda)$ with respect to λ_a in (3.2).

Let us consider an arbitrary function $\mathcal{F}(\Gamma)$ of phase-space variables expandable as a series in powers of Γ^p . Because of the nilpotency $\Delta\Gamma^{p_1} \cdots \Delta\Gamma^{p_n} \equiv 0$, $n \geq 3$, the function $\mathcal{F}(\Gamma)$ under the transformations (3.3) can be expanded as

$$\mathcal{F}(\Gamma + \Delta\Gamma) = \mathcal{F}(\Gamma) + \frac{\partial \mathcal{F}(\Gamma)}{\partial \Gamma^p} \Delta\Gamma^p + \frac{1}{2} \frac{\partial^2 \mathcal{F}(\Gamma)}{\partial \Gamma^p \partial \Gamma^q} \Delta\Gamma^q \Delta\Gamma^p. \quad (3.4)$$

Let the function $\mathcal{F}(\Gamma)$ be now invariant with respect to infinitesimal BRST-antiBRST transformations (2.8),

$$s^a \mathcal{F}(\Gamma) = 0, \quad \text{where} \quad s^a \mathcal{F}(\Gamma) = \frac{\partial \mathcal{F}(\Gamma)}{\partial \Gamma^p} s^a \Gamma^p, \quad (3.5)$$

and introduce *finite BRST-antiBRST transformations* in generalized Hamiltonian formalism as invariance transformations of the function $\mathcal{F}(\Gamma)$ under finite transformations of the variables Γ^p , such that

$$\mathcal{F}(\Gamma + \Delta\Gamma) = \mathcal{F}(\Gamma), \quad \Delta\Gamma^p \frac{\overleftarrow{\partial}}{\partial \lambda_a} \Big|_{\lambda=0} = s^a \Gamma^p \quad \text{and} \quad \Delta\Gamma^p \frac{\overleftarrow{\partial}}{\partial \lambda_a} \frac{\overleftarrow{\partial}}{\partial \lambda_b} = -\frac{1}{2} \varepsilon^{ab} s^2 \Gamma^p, \quad \text{where} \quad s^2 \equiv s_a s^a. \quad (3.6)$$

Namely, for the transformed variables $\check{\Gamma}^p = \Gamma^p + \Delta\Gamma^p$ we have²

$$\check{\Gamma}^p = \Gamma^p \left(1 + \overleftarrow{s}^a \lambda_a + \frac{1}{4} \overleftarrow{s}^2 \lambda^2 \right), \quad \text{or, equivalently,} \quad \Delta\Gamma^p = (s^a \Gamma^p) \lambda_a + \frac{1}{4} (s^2 \Gamma^p) \lambda^2, \quad \text{where} \quad \overleftarrow{s}^2 \equiv \overleftarrow{s}^a \overleftarrow{s}_a, \quad (3.7)$$

which is realized on phase-space trajectories $\Gamma^p(t)$ as follows:

$$\check{\Gamma}^p(t) = \Gamma^p(t) \left(1 + \overleftarrow{s}^a \lambda_a + \frac{1}{4} \overleftarrow{s}^2 \lambda^2 \right), \quad \text{or, equivalently,} \quad \Delta\Gamma^p(t) = (s^a \Gamma^p)(t) \lambda_a + \frac{1}{4} (s^2 \Gamma^p)(t) \lambda^2. \quad (3.8)$$

Let us now consider an arbitrary functional of the phase-space variables, $F(\Gamma)$, expandable as a series in powers of Γ^p . Under the transformations (2.9), the functional $F(\Gamma)$ can be presented as

$$F(\Gamma + \Delta\Gamma) = F(\Gamma) + \int dt \frac{\delta F(\Gamma)}{\delta \Gamma^p(t)} \Delta\Gamma^p(t) + \frac{1}{2} \int dt' dt'' \frac{\delta^2 F(\Gamma)}{\delta \Gamma^p(t') \delta \Gamma^q(t'')} \Delta\Gamma^q(t'') \Delta\Gamma^p(t'). \quad (3.9)$$

By analogy with the definition (3.5) of BRST-antiBRST transformations of functions, we let the functional $F(\Gamma)$ be invariant with respect to infinitesimal BRST-antiBRST transformations for trajectories (2.9),

$$s^a F(\Gamma) = 0, \quad \text{where} \quad s^a F(\Gamma) = \int dt \frac{\delta F(\Gamma)}{\delta \Gamma^p(t)} (s^a \Gamma^p)(t), \quad (3.10)$$

and introduce the *finite BRST-antiBRST transformations of functionals* as invariance transformations of a functional $F(\Gamma)$ under finite transformations of trajectories $\Gamma^p(t) \rightarrow \check{\Gamma}^p(t)$, such that

$$F(\check{\Gamma}) = F(\Gamma), \quad \check{\Gamma}^p(t) = \Gamma^p(t) \left(1 + \overleftarrow{s}^a \lambda_a + \frac{1}{4} \overleftarrow{s}^2 \lambda^2 \right). \quad (3.11)$$

The definitions of finite BRST-antiBRST transformations realized on functions (3.8) and functionals (3.11) are consistent. Indeed, for an arbitrary function $\mathcal{F}(t) = \mathcal{F}(\Gamma(t))$ with the corresponding functional $F(\Gamma) = \int dt \mathcal{F}(t)$, we have

$$s^a F(\Gamma) = \int dt \frac{\delta F(\Gamma)}{\delta \Gamma^p(t)} (s^a \Gamma^p)(t) = \int dt \frac{\partial \mathcal{F}(t)}{\partial \Gamma^p(t)} (s^a \Gamma^p)(t) = \int dt s^a \mathcal{F}(t) \quad (3.12)$$

$$\implies \Delta F(\Gamma) = \int dt \Delta \mathcal{F}(\Gamma(t)), \quad \text{with} \quad \Delta F(\Gamma) = F(\check{\Gamma}) - F(\Gamma), \quad \Delta \mathcal{F}(\Gamma(t)) = \mathcal{F}(\check{\Gamma}(t)) - \mathcal{F}(\Gamma(t)). \quad (3.13)$$

²As shown in [17], the validity of the algebra of BRST-antiBRST transformations for its generators $\overleftarrow{s}^a \overleftarrow{s}^b + \overleftarrow{s}^b \overleftarrow{s}^a = 0$, realized in an appropriate space of variables in Lagrangian [14] and generalized Hamiltonian formalism [12] allows one to restore the finite group form $\check{\Gamma} - \Gamma = \Gamma (\overleftarrow{s}^a \lambda_a + (1/4) \overleftarrow{s}^2 \lambda^2)$, or, identically, $\check{\Gamma} = \Gamma (1 + \overleftarrow{s}^a \lambda_a + (1/4) \overleftarrow{s}^2 \lambda^2) = \Gamma \exp(\overleftarrow{s}^a \lambda_a)$. Equivalently, the realization of the generators in terms of odd-valued anticommuting vector fields, $\overleftarrow{s}^a(\Gamma) = \frac{\overleftarrow{\partial}}{\partial \Gamma^p} (\Gamma^p \overleftarrow{s}^a)$, due to the Frobenius theorem, leads to the same form of finite BRST-antiBRST transformations.

Formula (3.12) describes the rule according to which the generators³ s^a of BRST-antiBRST transformations act on functionals via functions given in the phase space of Γ^p .

The consistency of definitions (3.7), (3.8), (3.11) is readily established by considering the respective equations $\Delta\mathcal{F} = 0$, $\Delta\mathcal{F}(t) = 0$, $\Delta F = 0$. For the first equation, we have

$$\frac{\partial\mathcal{F}(\Gamma)}{\partial\Gamma^p} \left[(s^a\Gamma^p)\lambda_a + \frac{1}{4} (s^2\Gamma^p)\lambda^2 \right] + \frac{1}{2} \frac{\partial^2\mathcal{F}(\Gamma)}{\partial\Gamma^p\partial\Gamma^q} \left[(s^a\Gamma^q)\lambda_a + \frac{1}{4} (s^2\Gamma^q)\lambda^2 \right] \left[(s^b\Gamma^p)\lambda_b + \frac{1}{4} (s^2\Gamma^p)\lambda^2 \right] = 0. \quad (3.14)$$

Taking into account the fact that $\lambda_a\lambda^2 = \lambda^4 \equiv 0$, the invariance relations $s^a F(\Gamma) = (\partial\mathcal{F}/\partial\Gamma^p) s^a\Gamma^p = 0$, and their differential consequence (after applying s^b and multiplying by $\lambda_b\lambda_a$)

$$\frac{\partial^2\mathcal{F}(\Gamma)}{\partial\Gamma^p\partial\Gamma^q} (s^b\Gamma^q)\lambda_b (s^a\Gamma^p)\lambda_a = -\frac{1}{2} \frac{\partial\mathcal{F}(\Gamma)}{\partial\Gamma^p} (s^2\Gamma^p)\lambda^2, \quad (3.15)$$

in view of the definition (2.8) and properties (2.10), we find that the above equation (3.14) is satisfied identically:

$$\frac{\partial\mathcal{F}(\Gamma)}{\partial\Gamma^p} (s^a\Gamma^p)\lambda_a + \frac{1}{4} \frac{\partial\mathcal{F}(\Gamma)}{\partial\Gamma^p} (s^2\Gamma^p)\lambda^2 + \frac{1}{2} \frac{\partial^2\mathcal{F}(\Gamma)}{\partial\Gamma^p\partial\Gamma^q} (s^b\Gamma^q)\lambda_b (s^a\Gamma^p)\lambda_a \stackrel{(3.15)}{=} 0. \quad (3.16)$$

In a similar way, one can readily establish the consistency of definitions (3.8) and (3.11).

We can see that the finite variation $\Delta\Gamma^p$ includes the generators of BRST-antiBRST transformations (s^1, s^2) , as well as their commutator $s^2 = \varepsilon_{ab}s^b s^a = s^1 s^2 - s^2 s^1$. According to (3.6), (3.9) and $\lambda_a\lambda^2 = \lambda^4 \equiv 0$, the variations $\Delta\mathcal{F}(\Gamma)$, $\Delta F(\Gamma)$ of an arbitrary function $\mathcal{F}(\Gamma)$ and of an arbitrary functional $F(\Gamma)$ under the corresponding finite BRST-antiBRST transformations (3.7), (3.11) are given by

$$\Delta\mathcal{F} = (s^a\mathcal{F})\lambda_a + \frac{1}{4} (s^2\mathcal{F})\lambda^2 \quad \text{and} \quad \Delta F = (s^a F)\lambda_a + \frac{1}{4} (s^2 F)\lambda^2. \quad (3.17)$$

In particular, the functions Ω^a and \mathcal{H} obey finite BRST-antiBRST invariance:

$$\Delta\Omega^a = \{\Omega^a, \Omega^b\}\lambda_b + \frac{1}{4}\varepsilon_{bc}\{\Omega^a, \{\Omega^b, \Omega^c\}\}\lambda^2 = 0, \quad \Delta\mathcal{H} = \{\mathcal{H}, \Omega^a\}\lambda_a + \frac{1}{4}\varepsilon_{ab}\{\mathcal{H}, \{\Omega^a, \Omega^b\}\}\lambda^2 = 0, \quad (3.18)$$

due to the generating equations (2.5), with the corresponding property for the Hamiltonian action $S_H(\Gamma)$ in (2.12)

$$\Delta S_H(\Gamma) = S_H(\check{\Gamma}) - S_H(\Gamma) = \int dt \left[\frac{1}{2} \left(\check{\Gamma}^p \omega_{pq} \frac{d\check{\Gamma}^p}{dt} \right) (t) - H_\Phi(\check{\Gamma})(t) \right] - S_H(\Gamma) = \int dt \frac{d\mathcal{F}(t)}{dt}, \quad (3.19)$$

where we have used the finite BRST-antiBRST invariance (3.18) of the unitarizing Hamiltonian H_Φ and the following transformations of the term $(1/2) \int dt (\Gamma^p \omega_{pq} \dot{\Gamma}^q)$ with respect to the BRST-antiBRST transformations (3.8) of trajectories $\Gamma^p(t)$ leading to the appearance of $d\mathcal{F}(t)/dt$:

$$\frac{1}{2} \int dt \left(\check{\Gamma}^p \omega_{pq} \frac{d\check{\Gamma}^p}{dt} \right) (t) = \frac{1}{2} \left[(\Gamma^p \partial_p \Omega^a - 2\Omega^a)\lambda_a + \frac{1}{4} \Gamma^p s_a (\partial_p \Omega^a) \lambda^2 \right] \Big|_{t_{\text{in}}}^{t_{\text{out}}} + \frac{1}{2} \int dt (\Gamma^p \omega_{pq} \dot{\Gamma}^q) (t), \quad (3.20)$$

which reflects the equality of the action in terms of the new phase-space coordinates $\check{\Gamma}$ to the action in terms of the old coordinates Γ up to a total derivative. The parameters λ_a in (3.7), (3.8) and (3.11) may be constant, $\lambda_a = \text{const}$, as well as field-dependent, $\lambda_a = \lambda_a(\Gamma)$, thus determining *global* and *field-dependent finite BRST-antiBRST transformations*. At the same time, we emphasize that the parameters $\lambda_a(\Gamma)$ are not regarded as functions of time t , and therefore of phase-space variables Γ^p , namely,

$$\frac{d\lambda_a(\Gamma)}{dt} = \frac{\partial\lambda_a(\Gamma)}{\partial\Gamma^p} = 0; \quad \text{however,} \quad \frac{\delta\lambda_a(\Gamma)}{\delta\Gamma^p} \neq 0. \quad (3.21)$$

³To be more exact, one could use two different symbols for the generators s^a as they act on functions and functionals in (3.5), (3.10); however, in order to simplify the notation for virtually the same operation, in view of (3.12), we use the symbol s^a .

Relations (3.8) and (3.17) allow one to calculate the Jacobians of finite BRST-antiBRST transformations, as well as to investigate the group properties of finite BRST-antiBRST transformations, presented in respective Subsections 3.2, 3.4. Thus, the functional measure $d\Gamma$ in (2.3) turns out to be invariant with respect to the change of trajectories, $\Gamma^p(t) \rightarrow \check{\Gamma}^p(t)$, related to finite BRST-antiBRST transformations (3.7) with constant parameters λ_a . This is nothing else than Liouville's theorem for the transformations (3.7), being canonical, due to the identity

$$\check{P}_A d\check{Q}^A - \check{H}_\Phi(\check{P}, \check{Q}) dt = P_A dQ^A - H_\Phi(P, Q) dt + dF, \quad (3.22)$$

which takes place for the contact 1-form, as one makes the substitution $\Gamma \rightarrow \check{\Gamma}$, setting $\check{H}_\Phi(\check{\Gamma}) = H_\Phi(\check{\Gamma})$ and taking account of (3.19). The invariance of the measure, $d\check{\Gamma} = d\Gamma$, along with the invariance (3.19) of the action $S_H(\Gamma)$, justifies the term “finite BRST-antiBRST transformations” as applied to the invariance transformations (3.11) of the integrand for Z_Φ .

3.2 Jacobians

Let us examine the change of the integration measure $d\Gamma \rightarrow d\check{\Gamma}$ in (2.3) under the finite transformations of phase-space trajectories, $\Gamma_t^p \rightarrow \check{\Gamma}_t^p = \Gamma_t^p + \Delta\Gamma_t^p$, with $\Delta\Gamma_t^p \equiv \Delta\Gamma^p(t)$ given by (3.8),

$$d\check{\Gamma} = d\Gamma \text{Sdet} \left(\frac{\delta\check{\Gamma}}{\delta\Gamma} \right), \quad \text{Sdet} \left(\frac{\delta\check{\Gamma}}{\delta\Gamma} \right) = \text{Sdet}(\mathbb{I} + M) = \exp[\text{Str} \ln(\mathbb{I} + M)] \equiv \exp(\mathfrak{S}), \quad (3.23)$$

where the Jacobian $\exp(\mathfrak{S})$ has the form

$$\begin{aligned} \mathfrak{S} = \text{Str} \ln(\mathbb{I} + M) &= - \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{Str}(M^n), \quad \text{Str}(M^n) = (-1)^{\varepsilon_p} \int dt (M^n)_p^p(t, t), \\ \mathbb{I} &= \delta_q^p \delta(t' - t''), \quad (M)_q^p(t', t'') = \frac{\delta\Delta\Gamma^p(t')}{\delta\Gamma^q(t'')}, \quad (AB)_q^p(t', t'') = \int dt (A)_r^p(t', t) B_q^r(t, t''). \end{aligned} \quad (3.24)$$

In the case of finite transformations corresponding to $\lambda_a = \text{const}$, the integration measure remains invariant (for details, see (A.9) in Appendix A)

$$\mathfrak{S}(\Gamma) = 0 \implies \left[\text{Sdet} \left(\frac{\delta\check{\Gamma}}{\delta\Gamma} \right) = 1, \quad d\check{\Gamma} = d\Gamma \right]. \quad (3.25)$$

As we turn to finite field-dependent transformations, $\lambda_a = \lambda_a(\Gamma)$, let us examine the particular case of functionally-dependent parameters⁴

$$\lambda_a(\Gamma) = \int dt (s_a \Lambda)(t) = \varepsilon_{ab} \int dt \{ \Lambda(t), \Omega^b(t) \}_t, \quad (3.26)$$

with a certain even-valued potential function $\Lambda(t) = \Lambda(\Gamma(t))$, which is inspired by field-dependent BRST-antiBRST transformations with the parameters (2.13). In this case, the integration measure takes the form (for details see (A.10) in Appendix A)

$$\mathfrak{S}(\Gamma) = -2 \ln[1 + f(\Gamma)], \quad f(\Gamma) = -\frac{1}{2} \int dt (s^2 \Lambda)_t, \quad (s^2 \Lambda)_t = \varepsilon_{ab} \{ \{ \Lambda, \Omega^a \}_t, \Omega^b \}_t, \quad (3.27)$$

$$d\check{\Gamma} = d\Gamma \exp \left[\frac{i}{\hbar} (-i\hbar \mathfrak{S}) \right] = d\Gamma \exp \left\{ \frac{i}{\hbar} \left[i\hbar \ln \left(1 - \frac{1}{2} \varepsilon_{ab} \int dt \{ \{ \Lambda, \Omega^a \}_t, \Omega^b \}_t \right)^2 \right] \right\}. \quad (3.28)$$

⁴The parameters λ_a are functionally-dependent, since $s^1 \lambda_1 + s^2 \lambda_2 = - \int dt s^2 \Lambda$.

3.3 Solution of the Compensation Equation

Let us apply the Jacobian (3.28) to cancel a change of the gauge Boson $\Phi(\Gamma)$ in (2.12):

$$\Phi \rightarrow \Phi + \Delta\Phi . \quad (3.29)$$

To this end, we subject $Z_{\Phi+\Delta\Phi}$ to a change of variables $\Gamma^p(t) \rightarrow \check{\Gamma}^p(t)$, given by (3.8) and parameterized by $\lambda_a(\Gamma)$ in accordance with (3.26). In terms of the new variables, we have

$$\begin{aligned} Z_{\Phi+\Delta\Phi} &= \int d\check{\Gamma} \exp \left[\frac{i}{\hbar} S_{H,\Phi+\Delta\Phi}(\check{\Gamma}) \right] = \int d\Gamma \exp [\Im(\Gamma)] \exp \left[\frac{i}{\hbar} S_{H,\Phi+\Delta\Phi}(\Gamma) \right] \\ &= \int d\Gamma \exp [\Im(\Gamma)] \exp \left\{ \frac{i}{\hbar} \left[S_{H,\Phi}(\Gamma) - \frac{1}{2} \varepsilon_{ab} \int dt \{ \{ \Delta\Phi(t), \Omega^a(t) \}_t, \Omega^b(t) \}_t \right] \right\} , \end{aligned} \quad (3.30)$$

using the transformation property (3.19) for $S_{H,\Phi+\Delta\Phi}$. If we now require the fulfillment of the relation

$$\exp [\Im(\Gamma)] = \exp \left[\frac{i}{2\hbar} \varepsilon_{ab} \int dt \{ \{ \Delta\Phi(t), \Omega^a(t) \}_t, \Omega^b(t) \}_t \right] , \quad (3.31)$$

which we will call the ‘‘compensation equation’’, then

$$Z_{\Phi+\Delta\Phi} = Z_{\Phi} . \quad (3.32)$$

Using the relation (3.28) and the compensation equation (3.31)

$$\frac{1}{2} \int dt \varepsilon_{ab} \{ \{ \Lambda, \Omega^a \}_t, \Omega^b \}_t = 1 - \exp \left[\frac{1}{4i\hbar} \varepsilon_{ab} \int dt \{ \{ \Delta\Phi(t), \Omega^a(t) \}_t, \Omega^b(t) \}_t \right] , \quad (3.33)$$

we can see that this is a functional equation for an unknown Bosonic function $\Lambda(\Gamma)$, which determines $\lambda_a(\Gamma)$ in accordance with $\lambda_a(\Gamma) = \int dt s_a \Lambda(\Gamma)$.

Introducing an auxiliary functional $y(\Gamma)$,

$$y(\Gamma) \equiv \frac{1}{4i\hbar} \varepsilon_{ab} \int dt \{ \{ \Delta\Phi(t), \Omega^a(t) \}_t, \Omega^b(t) \}_t = \frac{1}{4i\hbar} \Delta\hat{\Phi} \overleftarrow{s}^2 , \quad \text{where } \Delta\hat{\Phi} \equiv \int dt \Delta\Phi(t) , \quad (3.34)$$

which is BRST-antiBRST exact, $y(\Gamma) \overleftarrow{s}^a = 0$, and making use of $\overleftarrow{s}^2 = \overleftarrow{s}^a \overleftarrow{s}_a$, where $(F \overleftarrow{s}^a)(\Gamma)$ is identical with $s^a F(\Gamma)$ in (3.10), we present (3.33) in the form

$$\frac{1}{2} \int dt \Lambda \overleftarrow{s}^2 = 1 - \exp(y) = \frac{1}{4i\hbar} \left[g(y) \Delta\hat{\Phi} \right] \overleftarrow{s}^2 , \quad (3.35)$$

where $g(y) = [1 - \exp(y)]/y$ is a BRST-antiBRST exact functional. This provides an explicit solution of (3.35), with accuracy up to BRST-antiBRST exact terms:

$$\Lambda(\Gamma|\Delta\Phi) = \frac{1}{2i\hbar} g(y) \Delta\Phi . \quad (3.36)$$

Hence, the field-dependent parameters $\lambda_a(\Gamma)$ are implied by (3.26) and (3.36),

$$\lambda_a(\Gamma|\Delta\Phi) = \frac{1}{2i\hbar} g(y) \int dt (s_a \Delta\Phi)(t) = \frac{1}{2i\hbar} \varepsilon_{ab} g(y) \int dt \{ \Delta\Phi(t), \Omega^b(t) \}_t , \quad (3.37)$$

whereas the approximation linear in $\Delta\Phi$ follows from $g(0) = -1$,

$$\Lambda(\Gamma) = \frac{i}{2\hbar} \Delta\Phi + o(\Delta\Phi) \implies \lambda_a(\Gamma) = \frac{i}{2\hbar} \varepsilon_{ab} \int dt \{ \Delta\Phi(t), \Omega^b(t) \}_t + o(\Delta\Phi) , \quad (3.38)$$

and is identical with the parameters (2.13) of infinitesimal field-dependent BRST-antiBRST transformations.

3.4 Group Properties

The above relations (3.17)

$$\Delta \mathcal{F} = (s^a \mathcal{F}) \lambda_a + \frac{1}{4} (s^2 \mathcal{F}) \lambda^2, \quad \Delta F = (s^a F) \lambda_a + \frac{1}{4} (s^2 F) \lambda^2,$$

describing the finite variations of functions, $\mathcal{F} = \mathcal{F}(\Gamma(t))$, and functionals, $F = F(\Gamma)$, induced by finite BRST-antiBRST transformations, allow one to study the group properties of these transformations, with the provision that the transformations do not form neither a Lie superalgebra nor a vector superspace, due to the quadratic dependence on the parameters λ_a .

Let us study the composition of finite variations $\Delta_{(1)}\Delta_{(2)}$ acting on an object $A(\Gamma)$ being an arbitrary function or a functional. Using the Leibnitz-like properties of the generators of BRST-antiBRST transformations, s^a and s^2 , acting on the product of any functions (functionals) A, B with definite Grassmann parities,

$$\begin{aligned} s^a (AB) &= (s^a A) B (-1)^{\varepsilon_B} + A (s^a B), \quad s_a (AB) = (s_a A) B (-1)^{\varepsilon_B} + A (s_a B), \\ s^2 (AB) &= (s^2 A) B - 2 (s_a A) (s^a B) (-1)^{\varepsilon_B} + A (s^2 B), \end{aligned} \quad (3.39)$$

and the identities

$$s^a s^b = (1/2) \varepsilon^{ab} s^2 \quad \text{and} \quad s_a s^b = -s^b s_a = (1/2) \delta_a^b s^2 \quad \text{and} \quad s^a s^b s^c \equiv 0, \quad (3.40)$$

with the notation $UV \equiv U_a V^a = -U^a V_a$ for pairing up any $\text{Sp}(2)$ -vectors U^a, V^a , we obtain

$$\begin{aligned} s^a (\Delta A) &= s^a \left[(s^b A) \lambda_b + \frac{1}{4} (s^2 A) \lambda^2 \right] = s^a [(s^b A) \lambda_b] + (1/4) s^a [(s^2 A) \lambda^2] \\ &= - (s^a s^b A) \lambda_b + (s^b A) (s^a \lambda_b) + (1/4) (s^2 A) (s^a \lambda^2) \\ &= - (1/2) (s^2 A) \lambda^a - (s A) (s^a \lambda) + (1/4) (s^2 A) (s^a \lambda^2) \end{aligned} \quad (3.41)$$

and

$$\begin{aligned} s^2 (\Delta A) &= s^2 \left[(s^b A) \lambda_b + \frac{1}{4} (s^2 A) \lambda^2 \right] = s^2 [(s^b A) \lambda_b] + \frac{1}{4} s^2 [(s^2 A) \lambda^2] \\ &= 2 (s_a s^b A) (s^a \lambda_b) + (s^b A) (s^2 \lambda_b) + \frac{1}{4} (s^2 A) (s^2 \lambda^2) \\ &= - (s^2 A) (s \lambda) - (s A) (s^2 \lambda) + \frac{1}{4} (s^2 A) (s^2 \lambda^2). \end{aligned} \quad (3.42)$$

Therefore, $\Delta_{(1)}\Delta_{(2)}A$ is given by

$$\begin{aligned} \Delta_{(1)}\Delta_{(2)}A &= (s^a \Delta_{(2)}A) \lambda_{(1)a} + \frac{1}{4} (s^2 \Delta_{(2)}A) \lambda_{(1)}^2 \\ &= \left[- (1/2) (s^2 A) \lambda_{(2)}^a - (s A) (s^a \lambda_{(2)}) + (1/4) (s^2 A) (s^a \lambda_{(2)}^2) \right] \lambda_{(1)a} \\ &\quad + \frac{1}{4} \left[(s^2 A) (s \lambda_{(2)}) - (s A) (s^2 \lambda_{(2)}) + \frac{1}{4} (s^2 A) (s^2 \lambda_{(2)}^2) \right] \lambda_{(1)}^2 \\ &\equiv (s^a A) \vartheta_{(1,2)a} + \frac{1}{4} (s^2 A) \theta_{(1,2)}, \end{aligned} \quad (3.43)$$

for certain functionals $\vartheta_{(1,2)}^a(\Gamma)$ and $\theta_{(1,2)}(\Gamma)$, constructed from the parameters $\lambda_{(j)}^a$, for $j = 1, 2$, which are generally field-dependent, $\lambda_{(j)}^a = \lambda_{(j)}^a(\Gamma)$,

$$\vartheta_{(1,2)}^a = - (s \lambda_{(2)}^a) \lambda_{(1)} + \frac{1}{4} (s^2 \lambda_{(2)}^a) \lambda_{(1)}^2, \quad (3.44)$$

$$\theta_{(1,2)} = \left[2 \lambda_{(2)} - (s \lambda_{(2)}^2) \right] \lambda_{(1)} - \left[(s \lambda_{(2)}) - \frac{1}{4} (s^2 \lambda_{(2)}^2) \right] \lambda_{(1)}^2. \quad (3.45)$$

Hence, the commutator of finite variations reads

$$[\Delta_{(1)}, \Delta_{(2)}] A = (s^a A) \vartheta_{[1,2]a} + \frac{1}{4} (s^2 A) \theta_{[1,2]} , \quad \vartheta_{[1,2]}^a \equiv \vartheta_{(1,2)}^a - \vartheta_{(2,1)}^a , \quad \theta_{[1,2]} \equiv \theta_{(1,2)} - \theta_{(2,1)} , \quad (3.46)$$

Finally, using the identity

$$\lambda_{(2)} \lambda_{(1)} - \lambda_{(1)} \lambda_{(2)} = \lambda_{(2)a} \lambda_{(1)}^a - \lambda_{(1)a} \lambda_{(2)}^a = \lambda_{(2)a} \lambda_{(1)}^a - \lambda_{(2)a} \lambda_{(1)}^a \equiv 0 , \quad (3.47)$$

we obtain

$$\vartheta_{[1,2]}^a = \left(s \lambda_{(1)}^a \right) \lambda_{(2)} - \left(s \lambda_{(2)}^a \right) \lambda_{(1)} - \frac{1}{4} \left[\left(s^2 \lambda_{(1)}^a \right) \lambda_{(2)}^2 - \left(s^2 \lambda_{(2)}^a \right) \lambda_{(1)}^2 \right] , \quad (3.48)$$

$$\begin{aligned} \theta_{[1,2]} &= \left[\left(s \lambda_{(1)}^2 \right) \lambda_{(2)} - \left(s \lambda_{(2)}^2 \right) \lambda_{(1)} \right] + \left[\left(s \lambda_{(1)} \right) \lambda_{(2)}^2 - \left(s \lambda_{(2)} \right) \lambda_{(1)}^2 \right] \\ &\quad + \frac{1}{4} \left[\left(s^2 \lambda_{(2)}^2 \right) \lambda_{(1)}^2 - \left(s^2 \lambda_{(1)}^2 \right) \lambda_{(2)}^2 \right] . \end{aligned} \quad (3.49)$$

where $\vartheta_{[1,2]}^a, \theta_{[1,2]}$ possess the symmetry properties $\vartheta_{[1,2]}^a = -\vartheta_{[2,1]}^a, \theta_{[1,2]} = -\theta_{[2,1]}$. In particular, assuming $A(\Gamma) = \Gamma^p$ in (3.46), we have

$$[\Delta_{(1)}, \Delta_{(2)}] \Gamma^p = (s^a \Gamma^p) \vartheta_{[1,2]a} + \frac{1}{4} (s^2 \Gamma^p) \theta_{[1,2]} . \quad (3.50)$$

In general, the commutator (3.50) of finite BRST-antiBRST transformations does not belong to the class of these transformations due to the opposite symmetry properties of $\vartheta_{[1,2]a} \vartheta_{[1,2]}^a$ and $\theta_{[1,2]}$,

$$\vartheta_{[1,2]a} \vartheta_{[1,2]}^a = \vartheta_{[2,1]a} \vartheta_{[2,1]}^a , \quad \theta_{[1,2]} = -\theta_{[2,1]} , \quad (3.51)$$

which implies that $\theta_{[1,2]} = \vartheta_{[1,2]a} \vartheta_{[1,2]}^a$ in (3.50) is possible only in the particular case $\theta_{[1,2]} = \vartheta_{[1,2]a} \vartheta_{[1,2]}^a = 0$. This reflects the fact that a finite nonlinear transformation has the form of a group element, i.e., not an element of a Lie superalgebra; however, the linear approximation $\Delta^{\text{lin}} \Gamma^p = (s^a \Gamma^p) \lambda_a$ to a finite transformation $\Delta \Gamma^p = \Delta^{\text{lin}} \Gamma^p + O(\lambda^2)$ does form an algebra; indeed, due to (3.46), (3.48), (3.49), we have

$$[\Delta_{(1)}^{\text{lin}}, \Delta_{(2)}^{\text{lin}}] A = \Delta_{[1,2]}^{\text{lin}} A = (s^a A) \lambda_{[1,2]a} , \quad \lambda_{[1,2]}^a \equiv \left(s_b \lambda_{(1)}^a \right) \lambda_{(2)}^b - \left(s_b \lambda_{(2)}^a \right) \lambda_{(1)}^b . \quad (3.52)$$

Thus, the construction of finite BRST-antiBRST transformations reduces to the usual BRST-antiBRST transformations, $\delta \Gamma^p = \Delta^{\text{lin}} \Gamma^p$, linear in the infinitesimal parameter $\mu_a = \lambda_a$, as one selects the approximation that forms an algebra with respect to the commutator.

Using the above results, let us now consider an operator \mathcal{U} , such that

$$\mathcal{U} A = A + \Delta A , \quad \text{where} \quad \Delta A = (s^a A) \lambda_a + \frac{1}{4} (s^2 A) \lambda^2 , \quad \Delta_{(1)} \Delta_{(2)} A = (s^a A) \vartheta_{(1,2)a} + \frac{1}{4} (s^2 A) \theta_{(1,2)} , \quad (3.53)$$

and study its composition properties, namely,

$$\begin{aligned} \mathcal{U}_{(1)} \mathcal{U}_{(2)} A &= \mathcal{U}_{(1)} (\mathcal{U}_{(2)} A) = \mathcal{U}_{(1)} (A + \Delta_{(2)} A) = A + \Delta_{(2)} A + \Delta_{(1)} (A + \Delta_{(2)} A) \\ &= A + \Delta_{(1)} A + \Delta_{(2)} A + \Delta_{(1)} \Delta_{(2)} A = A + s^a A [\lambda_{(1)a} + \lambda_{(2)a} + \vartheta_{(1,2)a}] + \frac{1}{4} s^2 A [\lambda_{(1)}^2 + \lambda_{(2)}^2 + \theta_{(1,2)}] , \end{aligned} \quad (3.54)$$

$$[\mathcal{U}_{(1)}, \mathcal{U}_{(2)}] A = [\Delta_{(1)}, \Delta_{(2)}] A = (s^a A) \vartheta_{[1,2]a} + \frac{1}{4} (s^2 A) \theta_{[1,2]} , \quad (3.55)$$

whence follows the explicit form of the operator \mathcal{U} , as well as the corresponding composition and commutator, in terms of the operator $\overleftarrow{\mathcal{U}}$, whose action is identical with that of \mathcal{U} :

$$\overleftarrow{\mathcal{U}}_{(1)} = 1 + \overleftarrow{s}^a \lambda_{(1)a} + \frac{1}{4} \overleftarrow{s}^2 \lambda_{(1)}^2 = \exp \{ \overleftarrow{s}^a \lambda_{(1)a} \} , \quad (3.56)$$

$$\overleftarrow{\mathcal{U}}_{(1,2)} \equiv \overleftarrow{\mathcal{U}}_{(1)} \overleftarrow{\mathcal{U}}_{(2)} = 1 + \overleftarrow{s}^a [\lambda_{(1)a} + \lambda_{(2)a} + \vartheta_{(2,1)a}] + \frac{1}{4} \overleftarrow{s}^2 [\lambda_{(1)}^2 + \lambda_{(2)}^2 + \theta_{(2,1)}] , \quad (3.57)$$

$$[\overleftarrow{\mathcal{U}}_{(1)}, \overleftarrow{\mathcal{U}}_{(2)}] = \overleftarrow{\mathcal{U}}_{(1,2)} - \overleftarrow{\mathcal{U}}_{(2,1)} = -\overleftarrow{s}^a \vartheta_{[1,2]a} - \frac{1}{4} \overleftarrow{s}^2 \theta_{[1,2]} , \quad (3.58)$$

with $\vartheta_{(1,2)a}$, $\theta_{(1,2)}$ and $\vartheta_{[1,2]a}$, $\theta_{[1,2]}$ given by (3.44), (3.45) and (3.48), (3.49). From the above, we can see that the set of the operators $\overleftarrow{\mathcal{U}} \equiv \overleftarrow{\mathcal{U}}(\lambda)$ forms an Abelian two-parametric Lie supergroup for constant odd-valued parameters λ , $\overleftarrow{\mathcal{U}}(\lambda_1) \overleftarrow{\mathcal{U}}(\lambda_2) = \overleftarrow{\mathcal{U}}(\lambda_2) \overleftarrow{\mathcal{U}}(\lambda_1) = \overleftarrow{\mathcal{U}}(\lambda_1 + \lambda_2)$, with the unit element $e = \overleftarrow{\mathcal{U}}(0)$, whereas in the case of field-dependent λ it follows from (3.56)–(3.58) that the set of $\overleftarrow{\mathcal{U}}(\lambda(\Gamma))$ forms a non-linear algebraic structure.

4 Ward Identities and Gauge Dependence Problem

We can now apply finite BRST-antiBRST transformations to derive modified Ward (Slavnov–Taylor) identities and to study the problem of gauge-dependence for the generating functional of Green’s functions (2.3). As compared to the partition function Z_Φ in (3.32), the functional $Z_\Phi(I)$ in the presence of external sources $I_p(t)$ should depend on a choice of the gauge Boson Φ ; however, in view of the equivalence theorem [37], this dependence is highly structured, so that physical quantities cannot “feel” gauge dependence.

Using (3.11), the relation (3.17) for functionals, and the relations (3.19), (3.20) for the action $S_{H,\Phi}$, we have

$$S_{H,\Phi}(\check{\Gamma}) = S_{H,\Phi}(\Gamma) \left(1 + \overleftarrow{s}^a \lambda_a + \frac{1}{4} \overleftarrow{s}^2 \lambda^2 \right), \quad (4.1)$$

where the operators \overleftarrow{s}^a act in accordance with (3.34). Then, using (4.1) and (3.20), we obtain the formula

$$S_{H,\Phi}(\check{\Gamma}) = S_{H,\Phi}(\Gamma) + \frac{1}{2} \left[(\Gamma^p \partial_p \Omega^a - 2\Omega^a) \lambda_a + \frac{1}{4} \Gamma^p s_a (\partial_p \Omega^a) \lambda^2 \right] \Big|_{t_{\text{in}}}^{t_{\text{out}}}. \quad (4.2)$$

In terms of \overleftarrow{s}^a , the functional Jacobian (3.28) has the form

$$\exp(\mathfrak{J}) = \left[1 - \frac{1}{2} \left(\int dt \Lambda(t) \right) \overleftarrow{s}^2 \right]^{-2}. \quad (4.3)$$

Let us subject (2.3) to a field-dependent BRST-antiBRST transformation of trajectories (3.8). Then, the relation (4.3) for the Jacobian and the properties (3.19), (4.2) of gauge invariance for the action allow one to obtain a *modified Ward (Slavnov–Taylor) identity*:

$$\begin{aligned} & \left\langle \left\{ 1 + \frac{i}{\hbar} \int dt I_p(t) \Gamma^p(t) \left(\overleftarrow{s}^a \lambda_a(\Lambda) + \frac{1}{4} \overleftarrow{s}^2 \lambda^2(\Lambda) \right) - \frac{1}{4} \left(\frac{i}{\hbar} \right)^2 \int dt dt' I_p(t) \Gamma^p(t) \overleftarrow{s}^a I_q(t') \Gamma^q(t') \overleftarrow{s}_a \lambda^2(\Lambda) \right\} \right. \\ & \quad \times \left. \left\{ 1 - \frac{1}{2} \left[\int dt \Lambda(t) \right] \overleftarrow{s}^2 \right\}^{-2} \right\rangle_{\Phi, I} = 1, \end{aligned} \quad (4.4)$$

where the symbol “ $\langle \mathcal{O} \rangle_{\Phi, I}$ ” for any quantity $\mathcal{O} = \mathcal{O}(\Gamma)$ denotes a source-dependent average expectation value corresponding to a gauge $\Phi(\Gamma)$, namely,

$$\langle \mathcal{O} \rangle_{\Phi, I} = Z_\Phi^{-1}(I) \int d\Gamma \mathcal{O} \exp \left\{ \frac{i}{\hbar} \left[S_{H,\Phi}(\Gamma) + \int dt I(t) \Gamma(t) \right] \right\}, \quad \text{with} \quad \langle 1 \rangle_{\Phi, I} = 1. \quad (4.5)$$

In (4.4), both $\Lambda(\Gamma)$ and $I_p(t)$ are arbitrary, so that, due to the explicit presence of $\Lambda(\Gamma)$ [which implies $\lambda_a(\Lambda)$], the modified Ward identity implicitly depends on a choice of the gauge Bosonic function $\Phi(\Gamma)$ for non-vanishing $I_p(t)$, according to (3.36), (3.37). Thus, the corresponding Ward identities for Green’s functions obtained by differentiating (4.4) with respect to sources contain functionals $\lambda_a(\Lambda)$ and their derivatives [implicitly, $\Phi(\Gamma)$] as weight functionals, as compared to the usual Ward identities for constant λ_a . Indeed, for $\lambda_a = \text{const}$ the identity (4.4) implies two independent Ward identities at the first degree in powers of λ_a ,

$$\left\langle \int dt I_p(t) \Gamma^p(t) \overleftarrow{s}^a \right\rangle_{\Phi, I} = 0,$$

which are identical with those of (2.11), as well as a new Ward identity at the second degree in powers of λ_a ,

$$\left\langle \int dt I_p(t) \Gamma^p(t) \left[\overleftarrow{s}^2 - \overleftarrow{s}^a \left(\frac{i}{\hbar} \right) \int dt' I_q(t') (\Gamma^q(t') \overleftarrow{s}_a) \right] \right\rangle_{\Phi, I} = 0 .$$

Substituting, instead of $\lambda_a(\Lambda)$ [and $\Lambda(\Gamma)$] in (4.4), the solution (3.37) [(3.36)] of the compensation equation (3.31), we obtain, according to the study of Section 3.3, the following relation:

$$\begin{aligned} Z_{\Phi+\Delta\Phi}(I) = Z_{\Phi}(I) & \left\{ 1 + \left\langle \frac{i}{\hbar} \int dt I_p(t) \left[(s^a \Gamma^p(t)) \lambda_a(\Gamma| - \Delta\Phi) + \frac{1}{4} (s^2 \Gamma^p(t)) \lambda^2(\Gamma| - \Delta\Phi) \right] \right. \right. \\ & \left. \left. - (-1)^{\varepsilon_q} \left(\frac{i}{2\hbar} \right)^2 \int dt dt' I_q(t') I_p(t) (s^a \Gamma^p(t)) (s_a \Gamma^q(t')) \lambda^2(\Gamma| - \Delta\Phi) \right\rangle \right\} , \end{aligned} \quad (4.6)$$

which extends the result (3.32) to non-vanishing external sources $I_p(t)$.

Following [11], let us now enlarge the generating functional $Z_{\Phi}(I)$ to an extended generating functional of Green's functions $Z_{\Phi}(I, \Gamma^*, \bar{\Gamma})$ by adding to the action $S_{H, \Phi}$ some new terms with external sources (antifields) $\Gamma_{pa}^*(t)$ for $a = 1, 2$ and $\bar{\Gamma}_p(t)$, $\varepsilon(\Gamma_{pa}^*) + 1 = \varepsilon(\bar{\Gamma}_p) = \varepsilon_p$, multiplied by the respective BRST-antiBRST variations $(s^a \Gamma^p)(t)$ and their commutator $(s^2 \Gamma^p)(t)$, namely,

$$Z_{\Phi}(I, \Gamma^*, \bar{\Gamma}) = \int d\Gamma \exp \left\{ \frac{i}{\hbar} \left[S_{H, \Phi}(\Gamma) + \int dt \left(\Gamma_{pa}^* s^a \Gamma^p - \frac{1}{2} \bar{\Gamma}_p s^2 \Gamma^p + I\Gamma \right) \right] \right\} , \quad \text{for } Z_{\Phi}(I, 0, 0) = Z_{\Phi}(I). \quad (4.7)$$

If we make in (4.7) a change of variables (trajectories) in the extended space $(\Gamma^p, \Gamma_{pa}^*, \bar{\Gamma}_p)$,

$$\begin{aligned} \Gamma^p(t) & \rightarrow \check{\Gamma}^p(t) = \Gamma^p(t) \left(1 + \overleftarrow{s}^a \lambda_a + \frac{1}{4} \overleftarrow{s}^2 \lambda^2 \right) , \\ \Gamma_{pa}^*(t) & \rightarrow \check{\Gamma}_{pa}^*(t) = \Gamma_{pa}^*(t) , \\ \bar{\Gamma}_p(t) & \rightarrow \check{\bar{\Gamma}}_p(t) = \bar{\Gamma}_p(t) - \varepsilon^{ab} \lambda_a \Gamma_{pb}^*(t) , \end{aligned} \quad (4.8)$$

for $I_p = 0$ with finite constant parameters λ_a , we find that the integrand in (4.7) remains the same, in view of $\overleftarrow{s}_a \overleftarrow{s}_b \overleftarrow{s}_c \equiv 0$ and due to $\Delta(\Gamma_{pa}^* s^a \Gamma^p + \frac{1}{2} \bar{\Gamma}_p s^2 \Gamma^p) = 0$, which implies that the transformations (4.8) are *extended BRST-antiBRST transformations* for the functional $Z_{\Phi}(I, \Gamma^*, \bar{\Gamma})$.

Making in (4.7) a change of variables, which corresponds only to BRST-antiBRST transformations $\Gamma^p(t) \rightarrow \check{\Gamma}^p(t)$ with an arbitrary functional $\lambda_a(\Gamma) = \int dt \Lambda(t) \overleftarrow{s}_a$ from (3.26), we obtain a *modified Ward identity* for $Z_{\Phi}(I, \Gamma^*, \bar{\Gamma})$:

$$\begin{aligned} \left\langle \left\{ 1 + \frac{i}{\hbar} \int dt \left[I_p \left(\Gamma^p \overleftarrow{s}^a \lambda_a(\Lambda) + \frac{1}{4} \Gamma^p \overleftarrow{s}^2 \lambda^2(\Lambda) \right) + \frac{1}{2} \varepsilon^{ab} \Gamma_{pb}^* (\Gamma^p \overleftarrow{s}^2) \lambda_a \right] + \frac{\varepsilon_{ab}}{4} \left(\frac{i}{\hbar} \right)^2 \int dt \left[I_p (\Gamma^p \overleftarrow{s}^a) \right. \right. \right. \\ \left. \left. \left. + \frac{1}{2} \varepsilon^{ac} \Gamma_{pc}^* (\Gamma^p \overleftarrow{s}^2) \right] \int dt' \left[I_q (\Gamma^q \overleftarrow{s}^b) + \frac{1}{2} \varepsilon^{bd} \Gamma_{qd}^* (\Gamma^q \overleftarrow{s}^2) \right] \lambda^2(\Lambda) \right\} \left\{ 1 - \frac{1}{2} \left[\int dt \Lambda(t) \right] \overleftarrow{s}^2 \right\}^{-2} \right\rangle_{\Phi, I, \Gamma^*, \bar{\Gamma}} = 1 , \end{aligned} \quad (4.9)$$

where the symbol " $\langle \mathcal{O} \rangle_{\Phi, I, \Gamma^*, \bar{\Gamma}}$ " for any $\mathcal{O} = \mathcal{O}(\Gamma)$ stands for a source-dependent average expectation value for a gauge $\Phi(\Gamma)$ in the presence of the antifields $\Gamma_{pa}^*, \bar{\Gamma}_p$, namely,

$$\begin{aligned} \langle \mathcal{O} \rangle_{\Phi, I, \Gamma^*, \bar{\Gamma}} & = Z_{\Phi}^{-1}(I, \Gamma^*, \bar{\Gamma}) \int d\Gamma \mathcal{O} \exp \left\{ \frac{i}{\hbar} \left[S_{H, \Phi}(\Gamma, \Gamma^*, \bar{\Gamma}) + \int dt I(t) \Gamma(t) \right] \right\} , \\ \text{with } S_{H, \Phi}(\Gamma, \Gamma^*, \bar{\Gamma}) & = S_{H, \Phi}(\Gamma) + \int dt \left(\Gamma_{pa}^* s^a \Gamma^p - \frac{1}{2} \bar{\Gamma}_p s^2 \Gamma^p \right) . \end{aligned} \quad (4.10)$$

We can see that the difference of (4.4) and (4.9) is in the definitions (4.5) and (4.10), as well as in the presence of the terms proportional to $(1/2) \varepsilon^{ab} \Gamma_{pb}^* (\Gamma^p \overleftarrow{s}^2)$ at the first and second degrees in powers of λ_a , except for the Jacobian.

For constant parameters λ_a , we deduce from (4.9)

$$\left\langle \int dt \left[I_p(t) \Gamma^p(t) \overleftarrow{s}^a + \frac{1}{2} \varepsilon^{ab} \Gamma_{pb}^*(t) (\Gamma^p(t) \overleftarrow{s}^2) \right] \right\rangle_{\Phi, I, \Gamma^*, \bar{\Gamma}} = 0 , \quad (4.11)$$

as well as a new Ward identity at the second degree in powers of λ_a :

$$\begin{aligned} & \left\langle \int dt I_p(t) \Gamma^p(t) \overleftarrow{s}^2 + \varepsilon_{ab} \left(\frac{i}{\hbar} \right) \int dt \left[I_p \Gamma^p \overleftarrow{s}^a + \frac{1}{2} \varepsilon^{ac} \Gamma_{pc}^* (\Gamma^p \overleftarrow{s}^2) \right] (t) \right. \\ & \times \left. \int dt' \left[I_q(\Gamma^q \overleftarrow{s}^b + \frac{1}{2} \varepsilon^{bd} \Gamma_{qd}^* (\Gamma^q \overleftarrow{s}^2)) \right] (t') \right\rangle_{\Phi, I, \Gamma^*, \bar{\Gamma}} = 0 . \end{aligned} \quad (4.12)$$

The respective identities (4.11) and (4.12) may be represented as

$$\int dt \left[I_p(t) \frac{\overrightarrow{\delta}}{\delta \Gamma_{pa}^*(t)} - \varepsilon^{ab} \Gamma_{pb}^*(t) \frac{\overrightarrow{\delta}}{\delta \bar{\Gamma}_p(t)} \right] \ln Z_{\Phi}(I, \Gamma^*, \bar{\Gamma}) = 0 , \quad (4.13)$$

and

$$\varepsilon_{ab} \int dt dt' \left[I_p(t) \frac{\overrightarrow{\delta}}{\delta \Gamma_{pa}^*(t)} - \varepsilon^{ac} \Gamma_{pc}^*(t) \frac{\overrightarrow{\delta}}{\delta \bar{\Gamma}_p(t)} \right] \left[I_q(t') \frac{\overrightarrow{\delta}}{\delta \Gamma_{qb}^*(t')} - \varepsilon^{bd} \Gamma_{qd}^*(t') \frac{\overrightarrow{\delta}}{\delta \bar{\Gamma}_q(t')} \right] \ln Z_{\Phi}(I, \Gamma^*, \bar{\Gamma}) = 0 ,$$

being a differential consequence of (4.13) which follows from applying to the latter the operators

$$\int dt' \left[I_q(t') \frac{\overrightarrow{\delta}}{\delta \Gamma_{qb}^*(t')} - \varepsilon^{bd} \Gamma_{qd}^*(t') \frac{\overrightarrow{\delta}}{\delta \bar{\Gamma}_q(t')} \right] .$$

Let us consider the functional $S(\Gamma, \Gamma^*, \bar{\Gamma})$ being a functional Legendre transform of $\ln Z_{\Phi}(I, \Gamma^*, \bar{\Gamma})$ with respect to the sources $I_p(t)$:

$$\Gamma^p = \frac{\hbar}{i} \frac{\overrightarrow{\delta}}{\delta I_p} \ln Z_{\Phi}(I, \Gamma^*, \bar{\Gamma}) , \quad (4.14)$$

$$S(\Gamma, \Gamma^*, \bar{\Gamma}) = \frac{\hbar}{i} \ln Z_{\Phi}(I, \Gamma^*, \bar{\Gamma}) - \int dt I_p(t) \Gamma^p(t) , \quad (4.15)$$

$$\text{where } I_p(t) = -S(\Gamma, \Gamma^*, \bar{\Gamma}) \frac{\overleftarrow{\delta}}{\delta \Gamma^p(t)} . \quad (4.16)$$

From (4.13)–(4.16), we obtain an $\text{Sp}(2)$ -doublet of independent Ward identities for $S(\Gamma, \Gamma^*, \bar{\Gamma})$,

$$\frac{1}{2} (S, S)^a + V^a S = 0 , \quad (4.17)$$

in terms of the $\text{Sp}(2)$ -doublets of extended antibrackets and operators V^a known from the $\text{Sp}(2)$ -covariant Lagrangian quantization [14, 15] for gauge theories:

$$(F, G)^a = \int dt F \left(\frac{\overleftarrow{\delta}}{\delta \Gamma^p(t)} \frac{\overrightarrow{\delta}}{\delta \Gamma_{pa}^*(t)} - \frac{\overleftarrow{\delta}}{\delta \Gamma_{pa}^*(t)} \frac{\overrightarrow{\delta}}{\delta \Gamma^p(t)} \right) G , \quad V^a = \varepsilon^{ab} \int dt \Gamma_{pb}^*(t) \frac{\overrightarrow{\delta}}{\delta \bar{\Gamma}_p(t)} . \quad (4.18)$$

5 Relating Different Hamiltonian Gauges in Yang–Mills Theories

In this section, we examine the Yang–Mills theory, given by the Lagrangian action

$$S_0(A) = -\frac{1}{4} \int d^D x F_{\mu\nu}^u F^{\mu\nu} , \quad \text{for } F_{\mu\nu}^u = \partial_\mu A_\nu^u - \partial_\nu A_\mu^u + f^{uvw} A_\mu^w A_\nu^v , \quad (5.1)$$

with the Lorentz indices $\mu, \nu = 0, 1, \dots, D-1$, the metric tensor $\eta_{\mu\nu} = \text{diag}(-, +, \dots, +)$, and the totally antisymmetric $su(N)$ structure constants f^{uvw} for $u, v, w = 1, \dots, N^2 - 1$.

Let us consider the given gauge theory in the BRST-antiBRST generalized Hamiltonian quantization [11, 12]. To this end, note that the corresponding dynamical system is described in the initial phase space η [$x^\mu = (t, \mathbf{x})$, $t = x^0$, $\mathbf{x} = (x^1, \dots, x^{D-1})$], with the spatial indices being denoted as k, l : $\mu = (0, k)$

$$\eta = (p_i, q^i) = (\Pi_k^u, A^{uk}) , \quad i = (k, u, \mathbf{x})$$

by the classical Hamiltonian $H_0(\eta)$

$$H_0 = \int d\mathbf{x} \left(-\frac{1}{2} \Pi_k^u \Pi^{uk} + \frac{1}{4} F_{kl}^u F^{ukl} \right) \quad (5.2)$$

and by the set of linearly-independent constraints $T_\alpha(\eta)$, $\alpha = (u, \mathbf{x})$,

$$T_\alpha \equiv T^u = D_k^{uv} \Pi^{vk} , \quad D_k^{uv} = \delta^{uv} \partial_k + f^{uvw} A_k^w , \quad (5.3)$$

with the following involution relations:

$$\{T^u(t), H_0(t)\} = 0 , \quad \{T^u(t, \mathbf{x}), T^v(t, \mathbf{y})\} = \int dz f^{uvw} T^w(t, \mathbf{z}) \delta(\mathbf{x} - \mathbf{z}) \delta(\mathbf{y} - \mathbf{z}) . \quad (5.4)$$

Hence, the structure coefficients V_α^β , $U_{\alpha\beta}^\gamma$ arising in (2.2) are given by [$\alpha = (u, \mathbf{x})$, $\beta = (v, \mathbf{y})$, $\gamma = (w, \mathbf{z})$]

$$V_\alpha^\beta = 0 , \quad U_{\alpha\beta}^\gamma \equiv U^{uvw} = f^{uvw} \delta(\mathbf{x} - \mathbf{z}) \delta(\mathbf{y} - \mathbf{z}) .$$

The extended phase space Γ of the given irreducible dynamical system has the form

$$\Gamma = (P_A, Q^A) = (\Pi_k^u, A^{uk}, \mathcal{P}_a^u, C^{ua}, \lambda^u, \pi^u) ,$$

where the Grassmann parity and the ghost number of the variables Γ read as follows:

$$\varepsilon(\Gamma) = (0, 0, 1, 1, 0, 0) , \quad \text{gh}(\Gamma) = (0, 0, (-1)^a, (-1)^{a+1}, 0, 0) .$$

The explicit form of the structure coefficients and of the extended phase space Γ allows one to construct explicit solutions [38, 39] to the generating equations (2.5) with the boundary conditions (2.6) for the functions \mathcal{H} , Ω^a , namely,

$$\begin{aligned} \mathcal{H} &= H_0 , \\ \Omega^a &= \int d\mathbf{x} \left(C^{ua} D_k^{uv} \Pi^{vk} + \varepsilon^{ab} \mathcal{P}_b^u \pi^u + \frac{1}{2} \mathcal{P}_b^w f^{wvu} C^{ua} C^{vb} \right. \\ &\quad \left. - \frac{1}{2} \lambda^w f^{wvu} C^{ua} \pi^v - \frac{1}{12} \lambda^w f^{wvu} f^{uts} C^{sa} C^{tb} C^{vc} \varepsilon_{bc} \right) . \end{aligned} \quad (5.5)$$

Using (5.5), let us consider the generating functional of Green's functions $Z(I)$, given by (2.3). To do so, we choose the following Bosonic gauge function Φ in the relation (2.4) for the unitarizing Hamiltonian H_Φ :

$$\Phi = \int d\mathbf{x} \left(-\frac{\alpha}{2} A_k^u A^{uk} + \frac{1}{2\alpha} \lambda^u \lambda^u - \frac{\beta}{2} \varepsilon_{ab} C^{ua} C^{ub} \right) . \quad (5.6)$$

The unitarizing Hamiltonian H_Φ in (2.12) has the form

$$H_\Phi(t) = \int d\mathbf{x} \left(-\frac{1}{2} \Pi_k^u \Pi^{uk} + \frac{1}{4} F_{kl}^u F^{ukl} \right) + \frac{1}{2} \varepsilon_{ab} \{ \Phi, \Omega^a \} , \Omega^b \} ,$$

where

$$\begin{aligned} \frac{1}{2}\varepsilon_{ab} \{ \{ \Phi, \Omega^a \}, \Omega^b \} = \int d\mathbf{x} \left[-\alpha \left(\frac{1}{2}\varepsilon_{ab} C^{ub} D_k^{\text{uv}} (\partial^k C^{ua}) + \partial_k A^{uk} \pi^u \right) \right. \\ \left. + \frac{1}{2\alpha} \left(\varepsilon^{ab} \mathcal{P}_a^u \mathcal{P}_b^u + 2\lambda^u \mathcal{P}_a^v f^{\text{vu}w} C^{wa} - 2\lambda^u D_k^{\text{uv}} \Pi^{\text{vk}} - \frac{1}{4} \lambda^u \lambda^v f^{\text{vt}w} f^{\text{wsu}} C^{sc} C^{\text{td}} \varepsilon_{dc} \right) \right. \\ \left. + \beta \left(\pi^u \pi^u - \frac{1}{24} f^{\text{vu}w} f^{\text{wts}} C^{sa} C^{\text{tc}} C^{ub} C^{\text{vd}} \varepsilon_{ab} \varepsilon_{cd} \right) \right] . \end{aligned} \quad (5.7)$$

Integrating in the functional integral (2.3) over the momenta Π_k^u , \mathcal{P}_a^u and assuming the corresponding sources to be equal to zero, we obtain, with allowance made for the notation [39]

$$A_0^u \equiv \alpha^{-1} \lambda^u , \quad B^u \equiv \pi^u , \quad (5.8)$$

the following representation for the generating functional of Green's functions (2.3) in the space of fields $\phi^A(t, \mathbf{x}) = (A^{u\mu}, B^u, C^{ua})(t, \mathbf{x})$ with the corresponding sources $J_A(t, \mathbf{x})$:

$$Z(J) = \int d\phi \exp \left\{ \frac{i}{\hbar} \left[S_0(\phi) + S_{\text{gf}}(A, B) + S_{\text{gh}}(A, C) + S_{\text{add}}(C) + \int dt J_A(t) \phi^A(t) \right] \right\} , \quad (5.9)$$

where the gauge-fixing term S_{gf} , the ghost term S_{gh} , and the interaction term S_{add} , quartic in C^{ua} , are given by

$$S_{\text{gf}} = \int d^D x \left[\alpha (\partial^\mu A_\mu^u) - \beta B^u \right] B^u , \quad S_{\text{gh}} = \frac{\alpha}{2} \int d^D x \left(\partial^\mu C^{ua} \right) D_\mu^{\text{uv}} C^{\text{vb}} \varepsilon_{ab} , \quad (5.10)$$

$$S_{\text{add}} = \frac{\beta}{24} \int d^D x \quad f^{\text{vu}w} f^{\text{wts}} C^{sa} C^{\text{tc}} C^{ub} C^{\text{vd}} \varepsilon_{ab} \varepsilon_{cd} , \quad (5.11)$$

which differs from the result of [39], corresponding to the choice $\beta = 0$ in (5.6), by the presence of the term quadratic in B^u and the term quartic in C^{ua} . The result of integration (5.9) is identical with the generating functional of Green's functions recently obtained in [17] by the Lagrangian BRST-antiBRST quantization of the Yang-Mills theory. This coincidence establishes the unitarity of the S -matrix in the Lagrangian approach of [17].

Let us examine the choice of α, β leading to R_ξ -like gauges. Namely, in view of the contribution S_{gf}

$$S_{\text{gf}} = \int d^D x \left[\alpha (\partial^\mu A_\mu^u) - \beta B^u \right] B^u , \quad (5.12)$$

we impose the conditions

$$\alpha = 1 , \quad \beta = -\frac{\xi}{2} . \quad (5.13)$$

Thus, the gauge-fixing function $\Phi_{(\xi)} = \Phi_{(\xi)}(\Gamma)$ corresponding to an R_ξ -like gauge can be chosen as

$$\Phi_{(\xi)} = \frac{1}{2} \int d\mathbf{x} \left(-A_k^u A^{uk} + \lambda^u \lambda^u + \frac{\xi}{2} \varepsilon_{ab} C^{ua} C^{ub} \right) , \quad \text{so that} \quad (5.14)$$

$$\Phi_{(0)} = \frac{1}{2} \int d\mathbf{x} \left(-A_k^u A^{uk} + \lambda^u \lambda^u \right) \quad \text{and} \quad \Phi_{(1)} = \frac{1}{2} \int d\mathbf{x} \left(-A_k^u A^{uk} + \lambda^u \lambda^u + \frac{1}{2} \varepsilon_{ab} C^{ua} C^{ub} \right) , \quad (5.15)$$

where the gauge-fixing function $\Phi_{(0)}$ induces the contribution $S_{\text{gf}}(A, B)$ to the quantum action that arises in the case of the Landau gauge $\partial^\mu A_\mu^u = 0$ for $(\alpha, \beta) = (1, 0)$ in (5.12), whereas the function $\Phi_{(1)}(A, C)$ corresponds to the Feynman (covariant) gauge $\partial^\mu A_\mu^u + (1/2) B^u = 0$ for $(\alpha, \beta) = (1, -1/2)$ in (5.12).

Let us find the parameters $\lambda_a = \int dt s_a \Lambda$ of a finite field-dependent BRST-antiBRST transformation that connects an R_ξ gauge with an $R_{\xi+\Delta\xi}$ gauge:

$$\Delta\Phi_{(\xi)} = \Phi_{(\xi+\Delta\xi)} - \Phi_{(\xi)} = \frac{\Delta\xi}{4} \varepsilon_{ab} \int d\mathbf{x} C^{ua} C^{ub} . \quad (5.16)$$

Choosing the solution (3.36) of the compensation equation (3.31) according to the choice $\Delta\Phi = -\Delta\Phi_{(\xi)}$, we have

$$\Lambda(\Gamma| - \Delta\Phi_{(\xi)}) = -\frac{1}{2i\hbar}g(y)\Delta\Phi_{(\xi)}, \quad g(y) = [1 - \exp(y)]/y, \quad y(\Gamma| - \Delta\Phi_{(\xi)}) = -\frac{1}{4i\hbar}\varepsilon_{ab} \int dt \{ \{ \Delta\Phi_{(\xi)}, \Omega^a \}, \Omega^b \}. \quad (5.17)$$

According to (5.7), we have

$$\frac{1}{2}\varepsilon_{ab} \{ \{ \Delta\Phi_{(\xi)}, \Omega^a \}, \Omega^b \} = -\frac{\Delta\xi}{2} \int d\mathbf{x} \left(\pi^u \pi^u - \frac{1}{24} f^{vw} f^{wts} C^{sa} C^{tc} C^{ub} C^{vd} \varepsilon_{ab} \varepsilon_{cd} \right), \quad (5.18)$$

which implies

$$y(\Gamma| - \Delta\Phi_{(\xi)}) = \frac{\Delta\xi}{2i\hbar} \int d^D x \left(\pi^u \pi^u - \frac{1}{24} f^{vw} f^{wts} C^{sa} C^{tc} C^{ub} C^{vd} \varepsilon_{ab} \varepsilon_{cd} \right), \quad (5.19)$$

and, due to (3.37), (5.5), (5.16), the corresponding parameters $\lambda_a(\Gamma| - \Delta\Phi_{(\xi)})$ have the form

$$\lambda_a(\Gamma| - \Delta\Phi_{(\xi)}) = -\frac{1}{2i\hbar}\varepsilon_{ab}g(y) \int dt \{ \Delta\Phi_{(\xi)}, \Omega^b \} = \frac{\Delta\xi}{4i\hbar}\varepsilon_{ab}g(y) \int d^D x \pi^u C^{ub} \quad (5.20)$$

and generate the transition from an R_ξ -like gauge to another R_ξ -like gauge corresponding to $\xi + \Delta\xi$.

For comparison, notice that in the Lagrangian approach of [17] the transition from an R_ξ -like gauge to an $R_{\xi+\Delta\xi}$ -like gauge is described by the finite BRST-antiBRST transformation

$$\Delta A_\mu^m = D_\mu^{mn} C^{na} \lambda_a - \frac{1}{2} \left(D_\mu^{mn} B^n + \frac{1}{2} f^{mnl} C^{la} D_\mu^{nk} C^{kb} \varepsilon_{ba} \right) \lambda^2, \quad (5.21)$$

$$\Delta B^m = -\frac{1}{2} \left(f^{mnl} B^l C^{na} + \frac{1}{6} f^{mnl} f^{lrs} C^{sb} C^{ra} C^{nc} \varepsilon_{cb} \right) \lambda_a, \quad (5.22)$$

$$\Delta C^{ma} = \left(\varepsilon^{ab} B^m - \frac{1}{2} f^{mnl} C^{la} C^{nb} \right) \lambda_b - \frac{1}{2} \left(f^{mnl} B^l C^{na} + \frac{1}{6} f^{mnl} f^{lrs} C^{sb} C^{ra} C^{nc} \varepsilon_{cb} \right) \lambda^2, \quad (5.23)$$

with the field-dependent parameters $\lambda_a = \lambda_a(\phi)$

$$\begin{aligned} \lambda_a &= \frac{\Delta\xi}{4i\hbar} \varepsilon_{ab} \int d^D x \left(B^n C^{nb} + \frac{1}{2} f^{nml} C^{lc} C^{mb} C^{nd} \varepsilon_{cd} \right) \\ &\times \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \left[\frac{1}{4i\hbar} \Delta\xi \int d^D y \left(B^u B^u - \frac{1}{24} f^{uwt} f^{trs} C^{se} C^{rp} C^{wg} C^{uq} \varepsilon_{eg} \varepsilon_{pq} \right) \right]^n. \end{aligned} \quad (5.24)$$

Concluding, note that a finite change $\Phi \rightarrow \Phi + \Delta\Phi$ of the gauge condition induces a finite change of a function $\mathcal{G}_\Phi(\Gamma)$ or a functional $G_\Phi(\Gamma)$, so that in the reference frame corresponding to the gauge $\Phi + \Delta\Phi$ it can be represented, according to (3.17), (3.37), as follows:

$$\mathcal{G}_{\Phi+\Delta\Phi} = \mathcal{G}_\Phi + (s^a \mathcal{G}_\Phi) \lambda_a(\Delta\Phi) + \frac{1}{4} (s^2 \mathcal{G}_\Phi) \lambda_a(\Delta\Phi) \lambda^a(\Delta\Phi), \quad (5.25)$$

which is an extension of the infinitesimal change $\mathcal{G}_\Phi \rightarrow \mathcal{G}_\Phi + \delta\mathcal{G}_\Phi$ induced by a variation of the gauge, $\Phi \rightarrow \Phi + \delta\Phi$,

$$\mathcal{G}_{\Phi+\delta\Phi} = \mathcal{G}_\Phi - \frac{i}{2\hbar} (s^a \mathcal{G}_\Phi) \left(\int dt s_a \delta\Phi(t) \right), \quad (5.26)$$

corresponding, in the particular case $\mathcal{G}_\Phi(\eta)$, to the gauge transformations

$$\delta\eta = \{ \eta, T_{\alpha_0} \} C^{\alpha_0 a} \int dt (s_a \delta\Phi)(t) \equiv \{ \eta, T_{\alpha_0} \} \zeta^{\alpha_0}, \quad \text{for} \quad \zeta^{\alpha_0} = C^{\alpha_0 a} \int dt (s_a \delta\Phi)(t), \quad (5.27)$$

which in Yang–Mills theories are given by functions $\zeta^u(t, \mathbf{x})$:

$$\delta\mathcal{G}_\Phi = \mathcal{G}_{\Phi+\delta\Phi} - \mathcal{G}_\Phi = \int d\mathbf{x} \frac{\delta\mathcal{G}_\Phi(t)}{\delta\eta(t, \mathbf{x})} \{ \eta(t, \mathbf{x}), T^u(t, \mathbf{x}) \} \zeta^u(t, \mathbf{x}), \quad \text{where} \quad \zeta^u(t, \mathbf{x}) = -\frac{i}{2\hbar} C^{ua}(t, \mathbf{x}) \int dt' (s_a \delta\Phi)(t'). \quad (5.28)$$

Due to the presence of the term with $s^2 \mathcal{G}_\Phi$ in the finite gauge variation of a function $\mathcal{G}_\Phi(\eta)$, depending on the classical phase-space coordinates η , the representation (5.25) is more general than that which would correspond to the generalized Hamiltonian scheme [7, 29], having a form similar to (5.28), and therefore also to (5.26).

6 Conclusion

In the present work, we have proposed the concept of finite BRST-antiBRST transformations for phase-space variables and trajectories in the $\text{Sp}(2)$ -covariant generalized Hamiltonian quantization [11, 12]. This concept is realized in the form (3.7), (3.8), being polynomial in powers of a constant $\text{Sp}(2)$ -doublet of anticommuting Grassmann parameters λ_a and leaving the integrand in the partition function for dynamical systems subject to first-class constraints invariant to all orders of the constant doublet λ_a . We have established the fact that the finite BRST-antiBRST transformations with a constant doublet λ_a are canonical transformations.

We have introduced finite field-dependent BRST-antiBRST transformations as polynomials in powers of the $\text{Sp}(2)$ -doublet of Grassmann-odd functionals $\lambda_a(\Gamma)$, depending on the entire set of phase-space variables for an arbitrary constrained dynamical system in the $\text{Sp}(2)$ -covariant generalized Hamiltonian quantization. In a special case of functionally-dependent λ_a , we have obtained modified Ward identities (4.4), depending on λ_a , and therefore also on a variation of the gauge Boson, which leads to Ward identities for Green's functions with an additional weight function constructed from λ_a , and allows one to study the problem of gauge dependence (4.6) and to obtain the standard Ward identities with constant λ_a . We have calculated the Jacobian (3.27), (3.28) corresponding to this change of variables, by using a special class of transformations with functionally-dependent parameters $\lambda_a(\Gamma) = \int dt s_a \Lambda(\Gamma)$ for a Grassmann-even function $\Lambda(\phi)$ and Grassmann-odd generators s_a of BRST-antiBRST transformations in Hamiltonian formalism.

In comparison with finite field-dependent BRST-BFV transformations [26] in the generalized Hamiltonian formalism [28, 29], where a change of the gauge corresponds to a unique (up to BRST-exact terms) field-dependent parameter, it is only functionally-dependent finite BRST-antiBRST transformations with $\lambda_a = \int dt s_a \Lambda(\Gamma(t)|\Delta\Phi)$ that are in one-to-one correspondence with $\Delta\Phi$. We have found in (3.36) a solution $\Lambda(\Delta\Phi)$ to the compensation equation (3.31) for an unknown function Λ generating an $\text{Sp}(2)$ -doublet λ_a in (3.37), in order to establish a relation between the partition functions Z_Φ and $Z_{\Phi+\Delta\Phi}$, with the respective action $S_{H,\Phi}$ in a certain gauge induced by a gauge Boson Φ and the action $S_{H,\Phi+\Delta\Phi}$ induced by a different gauge $\Phi + \Delta\Phi$. This makes it possible to investigate the problem of gauge-dependence for the generating functional $Z_\Phi(I)$ under a finite change of the gauge in the form (4.6), leading to the gauge-independence of the physical S -matrix.

In terms of the potential Λ which generates finite field-dependent BRST-antiBRST transformations, we have explicitly constructed (5.20) the parameters λ_a generating a change of the gauge in the path integral for Yang-Mills theories within a class of linear R_ξ -like gauges in Hamiltonian formalism, related to even-valued gauge-fixing functions $\Phi_{(\xi)}$, with $\xi = 0, 1$ corresponding to the respective Landau and Feynman (covariant) gauges in Hamiltonian formalism. We have established, after integrating over momenta in the Hamiltonian path integral for an arbitrary gauge Boson $\Phi_{(\xi)}$, that the result (5.9) is identical with the generating functional of Green's functions recently obtained in [17] by the Lagrangian BRST-antiBRST quantization of the Yang-Mills theory, which justifies the unitarity of the S -matrix in the Lagrangian approach of [17]. We have suggested an explicit rule (5.25) of calculating the value of an arbitrary function $\mathcal{G}_\Phi(\Gamma)$ given in a certain gauge induced by the Bosonic function Φ , by using any other gauge $\Phi + \Delta\Phi$ in terms of finite field-dependent BRST-antiBRST transformations with functionally-dependent parameters $\lambda_a(\Delta\Phi)$ in (3.37), constructed using a finite variation $\Delta\Phi$.

Notice that, upon submission of this work to arXiv, we became aware of the article [40], in which similar problems are discussed. As compared to our present work, the study of [40] deals with a calculation of the Jacobian for a change of variables given by BRST-antiBRST (BRST-BFV by the terminology of [40]) transformations with functionally independent field-dependent odd-valued parameters $\lambda_a(\Gamma)$, subsequently used to formulate a compensation equation, similar to (3.31), but having a 2×2 matrix form, which satisfies the condition of resolvability only for functionally-dependent parameters, $\lambda_a = \int dt s_a \Lambda(\Gamma(t)|\Delta\Phi)$, whose form was first announced in our work [17].

There are various directions for extending the results of the present work: the study of soft BRST-BFV and

BRST-antiBRST symmetry breaking in the respective generalized Hamiltonian formulations [7, 29] and [11, 12]; the study of the Gribov problem [20] in the BRST-BFV and BRST-antiBRST generalized Hamiltonian formulations and its relation to the Lagrangian description [17, 34]; the calculation of Jacobians corresponding to BRST-antiBRST transformations linear in finite field-dependent parameters, as well as transformations with polynomial (group-like) but not functionally-dependent parameters λ_a [leading to an essentially different representation for the Jacobian than the one in (3.28)], which is a substantial part of our current study [41]. The other problems from the above list are also planned to be examined in our forthcoming works.

Acknowledgments

The authors are grateful to R. Metsaev for useful remarks and to the participants of the International Conference QFTG'2014, Tomsk, July 28–August 3, 2014. The study was supported by the RFBR grant under Project No. 12-02-00121 and by the grant of Leading Scientific Schools of the Russian Federation under Project No. 88.2014.2. The work was also partially supported by the Ministry of Science of the Russian Federation, Grant No. 2014/223.

Appendix

A Calculation of Jacobians

In this Appendix, we present the calculation of the Jacobian (3.23), (3.24) induced in the functional integral (2.3) by finite BRST-antiBRST transformations of phase-space trajectories (3.8) with an $\text{Sp}(2)$ -doublet λ_a of anticommuting parameters, considered in the case $\lambda_a = \text{const}$ and in the case of functionals $\lambda_a(\Gamma)$ of a special form, $\lambda_a(\Gamma) = \int dt s_a \Lambda(\Gamma)$. To this end, let us choose the parameters of (3.8) in the most general form $\lambda_a = \lambda_a(\Gamma)$ and consider the even matrix M in (3.24) with the elements $M_q^p(t'|t'') \equiv M_{q|t',t''}^p$, $\varepsilon(M_{q|t',t''}^p) = \varepsilon_p + \varepsilon_q$,

$$\begin{aligned} M_{q|t',t''}^p &= \frac{\delta(\Delta\Gamma_{t'}^p)}{\delta\Gamma_{t''}^q} = U_{q|t',t''}^p + V_{q|t',t''}^p + W_{q|t',t''}^p, \quad V_{q|t',t''}^p = (V_1)_{q|t',t''}^p + (V_2)_{q|t',t''}^p, \\ U_{q|t',t''}^p &= X_{t'}^{pa} \frac{\delta\lambda_a}{\delta\Gamma_{t''}^q}, \quad (V_1)_{q|t',t''}^p = \lambda_a \frac{\delta X_{t'}^{pa}}{\delta\Gamma_{t''}^q} (-1)^{\varepsilon_p+1}, \quad (V_2)_{q|t',t''}^p = \lambda_a Y_{t'}^p \frac{\delta\lambda_a}{\delta\Gamma_{t''}^q} (-1)^{\varepsilon_p+1}, \quad W_{q|t',t''}^p = -\frac{1}{2}\lambda^2 \frac{\delta Y_{t'}^p}{\delta\Gamma_{t''}^q}, \end{aligned} \quad (\text{A.1})$$

where the functions $X_t^{pa} = X^{pa}(\Gamma(t))$ and $Y_t^p = Y^p(\Gamma(t))$ are given by

$$X_t^{pa} = (s^a \Gamma^{pa})_t, \quad Y_t^p = -\frac{1}{2}(s^2 \Gamma^p)_t = -\frac{1}{2}\varepsilon_{ab} \int dt' \frac{\delta X_t^{pa}}{\delta\Gamma_{t'}^q} X_{t'}^{Bb} \quad (\text{A.2})$$

and possess the properties

$$\int dt' \frac{\delta X_t^{pa}}{\delta\Gamma_{t'}^q} X_{t'}^{qb} = \varepsilon^{ab} Y_t^p, \quad \int dt' \frac{\delta Y_t^p}{\delta\Gamma_{t'}^q} X_{t'}^{qa} = 0, \quad \int dt \frac{\delta X_t^{pa}}{\delta\Gamma_t^p} = 0. \quad (\text{A.3})$$

Indeed, due to the anticommutativity, $s^a s^b + s^b s^a = 0$, and nilpotency, $s^a s^b s^c = 0$, of the generators s^a , we have

$$\int dt' \frac{\delta X_t^{pa}}{\delta\Gamma_{t'}^q} X_{t'}^{qb} = \int dt' \frac{\delta X_t^{pa}}{\delta\Gamma_{t'}^q} (s^b \Gamma^q)_{t'} = (s^a s^b \Gamma^p)_t = \varepsilon^{ab} Y_t^p, \quad (\text{A.4})$$

$$\int dt' \frac{\delta Y_t^p}{\delta\Gamma_{t'}^q} X_{t'}^{qa} = \int dt' \frac{\delta Y_t^p}{\delta\Gamma_{t'}^q} (s^a \Gamma^q)_{t'} = (s^a Y^p)_t = -\frac{1}{2}\varepsilon_{bc} s^a (s^b s^c \Gamma^p)_t = 0; \quad (\text{A.5})$$

Hamiltonian formalism	Lagrangian formalism
$\Gamma_t^p, \Delta\Gamma_t^p = (s^a\Gamma_t^p)\lambda_a + \frac{1}{4}(s^2\Gamma_t^p)\lambda^2$ $\frac{\delta(\Delta\Gamma_{t'}^p)}{\delta\Gamma_{t''}^p} = M_{q t',t''}^p$ $s^a\Gamma_t^p = X_t^{pa}, Y_t^p = -\frac{1}{2}\varepsilon_{ab}\int dt' \frac{\delta X_t^{pa}}{\delta\Gamma_{t'}^q} X_{t'}^{Bb}$ $\int dt' \frac{\delta X_t^{pa}}{\delta\Gamma_{t'}^q} X_{t'}^{qb} = \varepsilon^{ab}Y_t^p, \int dt' \frac{\delta Y_t^p}{\delta\Gamma_{t'}^q} X_{t'}^{qa} = \int dt \frac{\delta X_t^{pa}}{\delta\Gamma_t^p} = 0$ $M_{q t',t''}^p = U_{q t',t''}^p + V_{q t',t''}^p + W_{q t',t''}^p$ $V_{q t',t''}^p = (V_1)_{q t',t''}^p + (V_2)_{q t',t''}^p$ $(V_1)_{q t',t''}^p = \lambda_a \frac{\delta X_{t'}^{pa}}{\delta\Gamma_{t''}^q} (-1)^{\varepsilon_p+1}$ $(V_2)_{q t',t''}^p = \lambda_a Y_{t'}^p \frac{\delta\lambda_a}{\delta\Gamma_{t''}^q} (-1)^{\varepsilon_p+1}$ $U_{q t',t''}^p = X_{t'}^{pa} \frac{\delta\lambda_a}{\delta\Gamma_{t''}^q}, W_{q t',t''}^p = -\frac{1}{2}\lambda^2 \frac{\delta Y_{t'}^p}{\delta\Gamma_{t''}^q}$ $\text{Str}(V_1) = \text{Str}(UW) = 0, \text{Str}(V_2) = 2\text{Str}(W)$ $\lambda_a = \text{const} : U = V_2 = 0, \Im = 0$ $\lambda_a = \int dt s_a\Lambda(\Gamma(t)) :$ $U^2 = f \cdot U, VU = (1+f) \cdot V_2, f = -\frac{1}{2}\text{Str}(U)$ $\int dt \frac{\delta\lambda_b}{\delta\Gamma_t^p} X_t^{pa} = s^a\lambda_b = \delta_b^a f, f = \frac{1}{2}s^a\lambda_a = -\frac{1}{2}\int dt (s^2\Lambda)(t)$ $\Im = -2\ln(1+f)$	$\phi^A, \Delta\phi^A = (s^a\phi^A)\lambda_a + \frac{1}{4}(s^2\phi^A)\lambda^2, A = (p, t)$ $\frac{\delta(\Delta\phi^A)}{\delta\phi^B} = M_B^A, A = (p, t'), B = (q, t'')$ $s^a\phi^A = X^{Aa}, Y^A = -\frac{1}{2}\varepsilon_{ab}\frac{\delta X^{Aa}}{\delta\phi^B} X^{Bb}$ $\frac{\delta X^{Aa}}{\delta\phi^B} X^{Bb} = \varepsilon^{ab}Y^A, \frac{\delta Y^A}{\delta\phi^B} X^{Bb} = \frac{\delta X^{Aa}}{\delta\phi^A} = 0$ $M_B^A = P_B^A + Q_B^A + R_B^A$ $Q_B^A = (Q_1)_B^A + (Q_2)_B^A$ $(Q_1)_B^A = \lambda_a \frac{\delta X^{Aa}}{\delta\phi^B} (-1)^{\varepsilon_A+1}$ $(Q_2)_B^A = \lambda_a Y^A \frac{\delta\lambda_a}{\delta\phi^B} (-1)^{\varepsilon_A+1}$ $P_B^A = X^{Aa} \frac{\delta\lambda_a}{\delta\phi^B}, R_B^A = -\frac{1}{2}\lambda^2 \frac{\delta Y^A}{\delta\phi^B}$ $\text{Str}(Q_1) = \text{Str}(PR) = 0, \text{Str}(Q_2) = 2\text{Str}(R)$ $\lambda_a = \text{const} : P = Q_2 = 0, \Im = 0$ $\lambda_a = s_a\Lambda(\phi) :$ $P^2 = f \cdot P, QP = (1+f) \cdot Q_2, f = -\frac{1}{2}\text{Str}(P)$ $\frac{\delta\lambda_b}{\delta\phi^A} X^{Aa} = s^a\lambda_b = \delta_b^a f, f = \frac{1}{2}s^a\lambda_a = -\frac{1}{2}s^2\Lambda$ $\Im = -2\ln(1+f)$

Table 1: Correspondence of the matrix elements in Lagrangian and Hamiltonian formalisms.

besides, we have

$$\begin{aligned}
X_t^{pa} &= \{\Gamma^p, \Omega^a\}_t, \quad \Gamma^p = (P_A, Q^A), \\
X_{A|t}^a &= \{P_A, \Omega^a\}_t = (-1)^{\varepsilon_A+1} \frac{\partial\Omega^a}{\partial Q^A} \Big|_t, \quad X_t^{Aa} = \{Q^A, \Omega^a\}_t = \frac{\partial\Omega^a}{\partial P_A} \Big|_t, \\
\int dt \frac{\delta X_t^{pa}}{\delta\Gamma_t^p} &= \int dt \left[\frac{\delta X_A^a(t)}{\delta P_A(t)} + \frac{\delta X^{Aa}(t)}{\delta Q^A(t)} \right] = \delta(0) \int dt \left[(-1)^{\varepsilon_A+1} \frac{\partial}{\partial P_A} \left(\frac{\partial\Omega^a}{\partial Q^A} \right) + \frac{\partial}{\partial Q^A} \left(\frac{\partial\Omega^a}{\partial P_A} \right) \right]_t \\
&= \delta(0) \int dt \left[-\frac{\partial}{\partial Q^A} \left(\frac{\partial\Omega^a}{\partial P_A} \right) + \frac{\partial}{\partial Q^A} \left(\frac{\partial\Omega^a}{\partial P_A} \right) \right]_t \equiv 0.
\end{aligned} \tag{A.6}$$

Recall that the Jacobian $\exp(\Im)$ induced by the finite BRST-antiBRST transformation (3.8) with the corresponding matrix M in (A.1) is given by (3.24), namely,

$$\Im = \text{Str} \ln(\mathbb{I} + M) = - \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{Str}(M^n). \tag{A.7}$$

In order to calculate the Jacobian explicitly in the cases $\lambda_a = \text{const}$ and $\lambda_a = \int dt s_a\Lambda$, it is sufficient to use the above properties (A.3), the identities $\lambda_a\lambda^2 = \lambda^4 \equiv 0$, the definitions

$$(AB)_{q|t',t''}^p = \int dt (A)_{r|t',t}^p (B)_{q|t,t''}^r, \quad \text{Str}(A) = (-1)^{\varepsilon_p} \int dt (A)_{p|t,t}^p \tag{A.8}$$

and the property of supertrace

$$\text{Str}(AB) = \text{Str}(BA),$$

which takes place for any even matrices A, B . In this setting, the task of calculation is formally identical with the one carried out in our previous work [17] that deals with the calculation of Jacobians induced by finite BRST-antiBRST

transformations in the Lagrangian approach to the Yang–Mills type of theories. Since the corresponding reasonings and results of [17] in the Lagrangian formalism can be literally reproduced in the Hamiltonian formalism of the present work, we give them briefly in Table 1.

Therefore, the Jacobians $\exp(\mathfrak{S})$ corresponding to the cases $\lambda_a = \text{const}$ and $\lambda_a = \int dt s_a \Lambda(\Gamma(t))$ are given by

$$\lambda_a = \text{const} : \quad \mathfrak{S} = 0 , \quad (A.9)$$

$$\lambda_a(\Gamma) = \int dt s_a \Lambda(\Gamma(t)) : \quad \mathfrak{S} = -2 \ln(1+f) , \quad f = -\frac{1}{2} \int dt (s^2 \Lambda)_t . \quad (A.10)$$

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